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Stability theorem for stochastic differential equations driven by G-Brownian motion

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Abstract

In this paper, stability theorems for stochastic differential equations and backward stochastic differential equations driven by G-Brownian motion are obtained. We show the existence and uniqueness of solutions to forward-backward stochastic differential equations driven by G-Brownian motion. Stability theorem for forward-backward stochastic differential equations driven by G-Brownian motion is also presented.

1 Introduction

Consider a family of ordinary stochastic differential equations (SDEs for short) parameterized by $\varepsilon \geq 0$,

$$X_t^\varepsilon = x_0^\varepsilon + \int_0^t b^\varepsilon(s, X_s^\varepsilon)ds + \int_0^t \sigma^\varepsilon(s, X_s^\varepsilon)dW_s, t \in [0, T],$$

where W_t is classical Brownian motion. It is well known that the strong convergence of the coefficient in L^2 implies the strong convergence of the solutions, that is, if

$$x_0^\varepsilon \to x_0^0, \text{ as } \varepsilon \to 0,$$

and

$$E[\int_0^T (|b^\varepsilon(s, X_s^0) - b^0(s, X_s^0)|^2 + |\sigma^\varepsilon(s, X_s^0) - \sigma^0(s, X_s^0)|^2)ds] \to 0, \text{ as } \varepsilon \to 0,$$

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then under Lipschitz and other reasonable assumptions, their solutions also converge strongly in L^2,

$$\forall\ t \in [0,T], E[|X_t^\varepsilon - X_t^0|^2] \to 0, \text{ as } \varepsilon \to 0.$$

This result, known as the continuous dependence theorem, or the stability property, can be found in many standard textbooks of SDEs (e.g., see [16]).

Backward stochastic differential equations (BSDEs for short) driven by classical Brownian motion were introduced, in linear case, by Bismut [3] in 1973. In 1990, Pardoux and Peng considered general BSDEs (see [12]). Similar continuous dependence theorem for the case of backward stochastic differential equations was obtained by El Karoui, Peng and Quenez (1994) [6] and Hu and Peng (1997) [10].

As for the forward-backward equations, Antonelli [1] first studied these equations, and he gave the existence and uniqueness when the time duration T is sufficiently small. Using a PDE approach, Ma, Protter and Yong [11] gave the existence and uniqueness to a class of forward-backward SDEs in which the forward SDE is non-degenerate. In 1995, Hu and Peng [9] study the existence and uniqueness of the solutions to forward-backward stochastic differential equations without the non-degeneracy condition.

Motivated by uncertainty problems, risk measures and the superhedging in finance, Peng (2006, see [13]) has introduced the notion of sublinear expectation space, which is a generalization of classical probability space. Together with the notion of sublinear expectation, Peng also introduced the related G-normal distribution and G-Brownian motion. The expectation associated with G-Brownian motion is a sublinear expectation which is called G-expectation. The stochastic calculus with respect to the G-Brownian motion has been established by Peng in [13], [14] and [15]. Since these notions were introduced, many properties of G-Brownian motion have been studied by authors, for example, [5], [7], [8], [17]-[19], et al.

Therefore, the natural questions are: stability properties for stochastic differential equations and backward stochastic differential equations driven by G-Brownian motion are also true? How to obtain the existence and uniqueness of the solution of a forward-backward stochastic differential equations driven by G-Brownian motion? The goal of this paper is to study stability properties for stochastic differential equations driven by G-Brownian motion (G-SDEs for short) and backward stochastic differential equations driven by G-Brownian motion (G-BSDEs for short). Indeed, under Lipschitz or integral-Lipschitz condition and other reasonable assumptions, stability theorems for G-SDEs and G-BSDEs are obtained. Meanwhile, we also show the existence and uniqueness of the solution of a new type of forward-backward stochastic differential equations driven by G-Brownian motion.

This paper is organized as follows: in Section 2, we recall briefly some notions and properties about G-expectation and G-Brownian motion. In Section 3, we study the stability properties of G-SDEs, while Section 4, study the G-BSDEs case. At last, the existence and uniqueness of the solution of forward-backward stochastic differential equations driven by G-Brownian motion are obtained. Stability theorem for forward-backward stochastic differential equations driven by G-Brownian motion is also presented.

2 Preliminaries

In this section, we introduce some notations and preliminaries about sublinear expectations and G-Brownian motion, which will be needed in what follows. More details concerning this section may be found in [13], [14] and [15].

Let Ω be a given set and let $\mathcal{H}$ be a linear space of real valued bounded functions defined on Ω. We suppose that $\mathcal{H}$ satisfies $C \in \mathcal{H}$ for each constant C and $|X| \in \mathcal{H}$, if $X \in \mathcal{H}$.

Definition 2.1. A sublinear expectation $\mathbb{E}$ is a functional $\mathbb{E} : \mathcal{H} \to R$ satisfying

(i) Monotonicity: $\mathbb{E}[X] \geq \mathbb{E}[Y]$ if $X \geq Y$.

(ii) Constant preserving: $\mathbb{E}[C] = C$ for $C \in R$.

(iii) Sub-additivity: For each $X, Y \in \mathcal{H}, \mathbb{E}[X + Y] \leq \mathbb{E}[X] + \mathbb{E}[Y]$.

(iv) Positive homogeneity: $\mathbb{E}[\lambda X] = \lambda\mathbb{E}[X]$ for $\lambda \geq 0$.

The triple $(\Omega, \mathcal{H}, \mathbb{E})$ is called a sublinear expectation space. If (i) and (ii) are satisfied, $\mathbb{E}[\cdot]$ is called a nonlinear expectation and the triple $(\Omega, \mathcal{H}, \mathbb{E})$ is called a nonlinear expectation space.

From now on, we consider the following sublinear expectation space $(\Omega, \mathcal{H}, \mathbb{E})$: if $X_1, \cdots, X_n \in \mathcal{H}$, then $\varphi(X_1, \cdots, X_n) \in \mathcal{H}$ for each $\varphi \in C_{l,Lip}(\mathbb{R}^n)$, where $C_{l,Lip}(R^n)$ denotes the linear space of functions φ satisfying $|\varphi(x) - \varphi(y)| \leq C(1 + |x|^m + |y|^m)|x - y|$ for $x, y \in R^n$, some $C > 0, m \in \mathbb{N}$ depending on φ.

Definition 2.2. Let X and Y be two n-dimensional random vectors defined on nonlinear expectation spaces $(\Omega_1, \mathcal{H}_1, \mathbb{E}_1)$ and $(\Omega_2, \mathcal{H}_2, \mathbb{E}_2)$, respectively. They are called identically distributed, denoted by $X \overset{d}{=} Y$, if

$$\mathbb{E}_1[\varphi(X)] = \mathbb{E}_2[\varphi(Y)], \text{ for } \forall\varphi \in C_{l,Lip}(R^n).$$

Definition 2.3. In a nonlinear expectation space $(\Omega, \mathcal{H}, \mathbb{E})$, a random vector $Y \in \mathcal{H}^n$ is said to be independent from another random vector $X \in \mathcal{H}^m$ under $\mathbb{E}[\cdot]$, if

$$\mathbb{E}[\varphi(X, Y)] = \mathbb{E}[\mathbb{E}[\varphi(x, Y)]_{x=X}], \text{ for } \forall\varphi \in C_{l,Lip}(R^{m+n}).$$

$\bar{X}$ is called an independent copy of X if $\bar{X} \overset{d}{=} X$ and $\bar{X}$ is independent from X.

Definition 2.4 (G-normal distribution). In a sublinear expectation space $(\Omega, \mathcal{H}, \mathbb{E})$, a random variable $X \in \mathcal{H}$ with

$$\mathbb{E}[X^2] = \bar{\sigma}^2, -\mathbb{E}[-X^2] = \underline{\sigma}^2,$$

is said to be $N(0; [\underline{\sigma}^2, \bar{\sigma}^2])$-distributed, if for each $\bar{X} \in \mathcal{H}$ which is an independent copy of X we have

$$aX + b\bar{X} \overset{d}{=} \sqrt{a^2 + b^2}X, \quad \forall a, b \geq 0.$$

Definition 2.5 (G-Brownian motion). A process $\{B_t(\omega)\}_{t\geq 0}$ in a sublinear expectation space $(\Omega, \mathcal{H}, \mathbb{E})$, is called a G-Brownian motion if for each $n \in N$ and $0 \leq t_1 \leq \cdots \leq t_n < \infty, B_{t_1}, \cdots, B_{t_n} \in \mathcal{H}$ and the following properties are satisfied:
(i) $B_0(\omega) = 0$;
(ii) For each $t, s \geq 0$, the increment $B_{t+s} - B_t$ is $N(0; [\underline{\sigma}^2, \bar{\sigma}^2])$-distributed and is independent from $(B_{t_1}, \cdots, B_{t_n})$ for each $n \in N$ and $0 \leq t_1 \leq \cdots \leq t_n \leq t$.

We denote by $\Omega = C_0^d(R^+)$ the space of all R^d-valued continuous paths $(\omega_t)_{t\in R^+}$, with $\omega_0 = 0$, equipped with the distance $\rho(\omega^1, \omega^2) := \sum\limits_{i=1}^{\infty} 2^{-i}[(\max\limits_{t\in[0,i]} |\omega_t^1 - \omega_t^2|) \wedge 1]$. Considering the canonical process $B_t(\omega) = (\omega_t)_{t\geq 0}$. For each fixed $T > 0$, set $\Omega_T := \{\omega_{\cdot\wedge T} : \omega \in \Omega\}$ and

$$L_{ip}(\Omega_T) := \{\varphi(B_{t_1}, B_{t_2}, ..., B_{t_m}) : m \geq 1, t_1, ..., t_m \in [0, T], \varphi \in C_{l,Lip}(R^{d\times m})\},$$

and define $L_{ip}(\Omega) := \bigcup\limits_{n=1}^{\infty} L_{ip}(\Omega_n)$.

Let ξ be a G-normal distributed, or $N(0; [\sigma^2, 1])$-distributed random variable in a sublinear expectation space $(\widetilde{\Omega}, \widetilde{\mathcal{H}}, \widetilde{\mathbb{E}})$. We now introduce a sublinear expectation $\hat{\mathbb{E}}$ defined on $L_{ip}(\Omega)$ via the following procedure: for each $X \in L_{ip}(\Omega)$ with

$$X = \varphi(B_{t_1} - B_{t_0}, B_{t_2} - B_{t_1}, \cdots, B_{t_m} - B_{t_{m-1}}),$$

for some $\varphi \in C_{l,Lip}(R^{d\times m})$ and $0 = t_0 < t_1 < \cdots < t_m < \infty$, we set

$$\hat{\mathbb{E}}[\varphi(B_{t_1} - B_{t_0}, \cdots, B_{t_m} - B_{t_{m-1}})] := \widetilde{\mathbb{E}}[\varphi(\sqrt{t_1 - t_0}\xi_1, \cdots, \sqrt{t_m - t_{m-1}}\xi_m)],$$

where $(\xi_1, \cdots, \xi_m)$ is an m-dimensional G-normal distributed random vector in a sublinear expectation space $(\widetilde{\Omega}, \widetilde{\mathcal{H}}, \widetilde{\mathbb{E}})$ such that $\xi_i \overset{d}{=} N(0; [\sigma^2, 1])$ and such that ξ_{i+1} is independent from $(\xi_1, \cdots, \xi_i)$ for each $i = 1, \cdots, m$.

The related conditional expectation of $X = \varphi(B_{t_1} - B_{t_0}, B_{t_2} - B_{t_1}, ..., B_{t_m} - B_{t_{m-1}})$ under Ω_{t_j} is defined by

$$\begin{aligned}\hat{\mathbb{E}}[X|\Omega_{t_j}] &= \hat{\mathbb{E}}[\varphi(B_{t_1}, B_{t_2} - B_{t_1}, \cdots, B_{t_m} - B_{t_{m-1}})|\Omega_{t_j}] \\ &:= \psi(B_{t_1},, \cdots, B_{t_j} - B_{t_{j-1}}),\end{aligned}$$

where

$$\psi(x_1,\cdots,x_j)=\widetilde{\mathbb{E}}[\varphi(x_1,\cdots,x_j,\sqrt{t_{j+1}-t_j}\xi_{j+1},\cdots,\sqrt{t_m-t_{m-1}}\xi_m)].$$

Definition 2.6. The expectation $\hat{\mathbb{E}}[\cdot]:L_{ip}(\Omega)\rightarrow R$ defined through the above procedure is called G-expectation. The corresponding canonical process $(B_t)_{t\geq 0}$ in the sublinear expectation space $(\Omega,L_{ip}(\Omega),\hat{\mathbb{E}})$ is called a G-Brownian motion.

We denote by $L_G^p(\Omega_T),p\geq 1$, the completion of $L_{ip}(\Omega_T)$ under the norm $||X||_p:=(\hat{\mathbb{E}}[|X|^p])^{1/p}$. Similarly, denote $L_G^p(\Omega)$ is complete space of $L_{ip}(\Omega)$. We give some important properties about conditional G-expectation $\hat{\mathbb{E}}[\cdot|\Omega_t],t\in[0,T]$.

Proposition 2.1. The conditional expectation $\hat{\mathbb{E}}[\cdot|\Omega_t],t\in[0,T]$ holds for each $X,Y\in L_G^1(\Omega_t)$:

(i) If $X\geq Y$, then $\hat{\mathbb{E}}[X|\Omega_t]\geq\hat{\mathbb{E}}[Y|\Omega_t]$.

(ii) $\hat{\mathbb{E}}[\eta|\Omega_t]=\eta$, for each $t\in[0,\infty)$ and $\eta\in L_G^1(\Omega_t)$.

(iii) $\hat{\mathbb{E}}[X|\Omega_t]-\hat{\mathbb{E}}[Y|\Omega_t]\leq\hat{\mathbb{E}}[X-Y|\Omega_t]$.

(iv) $\hat{\mathbb{E}}[\eta X|\Omega_t]=\eta^+\hat{\mathbb{E}}[X|\Omega_t]+\eta^-\hat{\mathbb{E}}[-X|\Omega_t]$ for each bounded $\eta\in L_G^1(\Omega_t)$.

(v) $\hat{\mathbb{E}}[\hat{\mathbb{E}}[X|\Omega_t]|\Omega_s]=\hat{\mathbb{E}}[X|\Omega_{t\wedge s}]$, in particular, $\hat{\mathbb{E}}[\hat{\mathbb{E}}[X|\Omega_t]]=\hat{\mathbb{E}}[X]$.

Next, we introduce the Itô's integral with G-Brownian motion. For $T\in R^+$, a partition π_T of $[0,T]$ is a finite ordered subset $\pi_T=\{t_0,t_1,...,t_N\}$ such that $0=t_0<t_1<...<t_N=T$,

$$\mu(\pi_T):=\max\{|t_{i+1}-t_i|:i=0,1,...,N-1\}.$$

Using $\pi_T^N=\{t_0^N,t_1^N,...,t_N^N\}$ to denote a sequence of partitions of $[0,T]$ such that $\lim_{N\rightarrow\infty}\mu(\pi_T^N)=0$.

Let $p\geq 1$ be fixed. We consider the following type of simple processes: for a given partition $\pi_T=\{t_0,t_1,...,t_N\}$ of $[0,T]$, set $\eta_t(\omega)=\sum_{k=0}^{N-1}\xi_k(\omega)\mathbf{I}_{[t_k,t_{k+1})}(t)$, where $\xi_k\in L_G^p(\Omega_{t_k}),k=0,1,...,N-1$ are given. The collection of these processes is denoted by $M_G^{p,0}(0,T)$. For each $p\geq 1$, we denote by $M_G^p([0,T];R^n)$ the completion of $M_G^{p,0}([0,T];R^n)$ under the norm $||\eta_t||_{M_G^p([0,T])}:=(\int_0^T\hat{\mathbb{E}}[|\eta_t|^p]dt)^{\frac{1}{p}}$.

Definition 2.7. For an $\eta\in M_G^{p,0}(0,T)$, the related Bochner integral is

$$\int_0^T\eta_t(\omega)dt:=\sum_{k=0}^{N-1}\xi_k(\omega)(t_{k+1}-t_k).$$

Let $(B_t)_{t\geq 0}$ be a 1-dimensional G-Brownian motion with $G(a):=\frac{1}{2}\hat{\mathbb{E}}[aB_1^2]=\frac{1}{2}(\bar{\sigma}^2a^+-\underline{\sigma}^2a^-)$, where $\bar{\sigma}^2=\hat{\mathbb{E}}[B_1^2],\underline{\sigma}^2=-\hat{\mathbb{E}}[-B_1^2]$, $0\leq\underline{\sigma}\leq\bar{\sigma}<\infty$.

Definition 2.8. For an $\eta \in M_G^{2,0}(0,T)$ of the form $\eta_t(\omega) = \sum_{k=0}^{N-1} \xi_k(\omega)\mathbf{I}_{[t_k,t_{k+1})}(t)$, define

$$\int_0^T \eta(s)dB_s := \sum_{k=0}^{N-1} \xi_k(B_{t_{k+1}} - B_{t_k}).$$

Proposition 2.2. For each $\eta \in M_G^{2,0}(0,T)$, then

$$\hat{\mathbb{E}}[\int_0^T \eta(s)dB_s] = 0, \quad \hat{\mathbb{E}}[(\int_0^T \eta(s)dB_s)^2] \leq \bar{\sigma}^2 \int_0^T \hat{\mathbb{E}}[\eta^2(s)]ds.$$

Definition 2.9. For the 1-dimensional G-Brownian motion B_t, we denote $\langle B\rangle_t$ is the quadratic variation process of B_t, where $\langle B\rangle_t := \lim_{\mu(\pi_t^N)\to 0} \sum_{k=0}^{N-1} (B_{t_{k+1}^N} - B_{t_k^N})^2 = (B_t)^2 - 2\int_0^t B_s dB_s$.

Definition 2.10. For each $\eta \in M_G^{1,0}(0,T)$, define $\int_0^T \eta(s)d\langle B\rangle_s := \sum_{k=0}^{N-1} \xi_k(\langle B\rangle_{t_{k+1}} - \langle B\rangle_{t_k})$.

Proposition 2.3. For any $0 \leq t \leq T < \infty$,
(i) $\hat{\mathbb{E}}[|\int_0^T \eta_t d\langle B\rangle_t|] \leq \bar{\sigma}^2 \hat{\mathbb{E}}[\int_0^T |\eta_t| dt], \forall\ \eta_t \in M_G^1(0,T)$.
(ii) $\hat{\mathbb{E}}[(\int_0^T \eta_t dB_t)^2] = \hat{\mathbb{E}}[\int_0^T \eta_t^2 d\langle B\rangle_t], \forall\ \eta_t \in M_G^2(0,T)$.
(iii) $\hat{\mathbb{E}}[\int_0^T |\eta_t|^p dt] \leq \int_0^T \hat{\mathbb{E}}[|\eta_t|^p]dt, \forall\ \eta_t \in M_G^p(0,T),\ p \geq 1$.

3 Stability theorem of G-stochastic differential equations

In this section, we consider the stability theorem of G-stochastic differential equations. Consider the following stochastic differential equations driven by d-dimensional G-Brownian motion:

$$X_t = X_0 + \int_0^t b(s,X_s)ds + \sum_{i,j=1}^d \int_0^t h_{ij}(s,X_s)d\langle B^i, B^j\rangle_s + \\ + \sum_{j=1}^d \int_0^t \sigma_j(s,X_s)dB_s^j, \quad t \in [0,T],$$

the initial condition $X_0 \in R^n$, and b, h_{ij}, σ_j are given functions satisfying $b(\cdot,x), h_{ij}(\cdot,x),\ \sigma_j(\cdot,x) \in M_G^2([0,T];R^n)$ for each $x \in R^n$. Consider the following G-SDEs depending on a parameter $\varepsilon(\varepsilon \geq 0)$:

$$X_t^\varepsilon = X_0^\varepsilon + \int_0^t b^\varepsilon(s, X_s^\varepsilon)ds + \sum_{i,j=1}^d \int_0^t h_{ij}^\varepsilon(s, X_s^\varepsilon)d\langle B^i, B^j\rangle_s + \\ + \sum_{j=1}^d \int_0^t \sigma_j^\varepsilon(s, X_s^\varepsilon)dB_s^j, t \in [0,T].$$

We make the following assumptions:

Assumption 3.1. For any $\varepsilon \geq 0, x \in R^n, b^\varepsilon(\cdot, x), h_{ij}^\varepsilon(\cdot, x), \sigma_j^\varepsilon(\cdot, x) \in M_G^2([0,T]; R^n)$, $X_0^\varepsilon \in R^n$.

Assumption 3.2. For any $\varepsilon \geq 0, x, x_1, x_2 \in R^n$:

(H1) $|b^\varepsilon(t,x)|^2 + \sum\limits_{i,j=1}^d |h_{ij}^\varepsilon(t,x)|^2 + \sum\limits_{j=1}^d |\sigma_j^\varepsilon(t,x)|^2 \leq \alpha_1^2(t) + \alpha_2^2(t)|x|^2$,

(H2) $|b^\varepsilon(t,x_1) - b^\varepsilon(t,x_2)|^2 + \sum\limits_{i,j=1}^d |h_{ij}^\varepsilon(t,x_1) - h_{ij}^\varepsilon(t,x_2)|^2 + \sum\limits_{j=1}^d |\sigma_j^\varepsilon(t,x_1) - \sigma_j^\varepsilon(t,x_2)|^2 \leq \alpha^2(t)\rho(|x_1 - x_2|^2)$, where $\alpha_1 \in M_G^2([0,T]), \alpha_2 : [0,T] \to R^+$ and $\alpha : [0,T] \to R^+$ are Lebesgue integrable, and $\rho : (0,+\infty) \to (0,+\infty)$ is continuous, increasing, concave function satisfying $\rho(0+) = 0, \int_0^1 \frac{1}{\rho(r)}dr = +\infty$.

Assumption 3.3. (i) $\forall\ t \in [0,T]$, as $\varepsilon \to 0$,

$$\int_0^t \hat{\mathbb{E}}[|\phi^\varepsilon(s, X_s^0) - \phi^0(s, X_s^0)|^2]ds \to 0,$$

where $\phi = b, h_{ij}$ and σ_j, respectively, $i,j = 1, \cdots, d$.

(ii) As $\varepsilon \to 0$,

$$X_0^\varepsilon \to X_0^0.$$

Remark 3.1. The Assumptions 3.1 and 3.2 guarantee, for any $\varepsilon \geq 0$, the existence of a unique solution $X_t^\varepsilon \in M_G^2([0,T]; R^n)$ of G-SDEs (3.2)(see [2]), while the Assumption 3.3 will allow us to deduce the following stability theorem for G-SDEs.

Theorem 3.1. Under the Assumptions 3.1, 3.2 and 3.3, we have the following convergence: as $\varepsilon \to 0$,

$$\forall\ t \in [0,T], \quad \hat{\mathbb{E}}[|X_t^\varepsilon - X_t^0|^2] \to 0. \tag{1}$$

In order to prove Theorem 3.1, we need the following lemmas:

Lemma 3.1 (see Chemin and Lerner [4]). Let $\rho : (0,+\infty) \to (0,+\infty)$ be a continuous, increasing function satisfying $\rho(0+) = 0$, $\int_0^1 \frac{1}{\rho(r)}dr = +\infty$ and let u be a measurable, nonnegative function defined on $(0,+\infty)$ satisfying

$$u(t) \leq a + \int_0^t \beta(s)\rho(u(s))ds,\ t \in (0,+\infty),$$

where $a \in [0, +\infty)$, and $\beta : [0, T] \to R^+$ is Lebesgue integrable. Then
(i) if $a = 0$, then $u(t) = 0$, for $t \in [0, +\infty)$;
(ii) if $a > 0$, then

$$u(t) \leq v^{-1}(v(a) + \int_0^t \beta(s)ds),$$

where $v(t) := \int_{t_0}^t \frac{1}{\rho(s)} ds, t_0 \in (0, +\infty)$.
Lemma 3.2 (see Peng [15]). Let $\rho : R \to R$ be a continuous increasing, concave function defined on R, then for each $X \in L_G^1(\Omega), \forall\, t \geq 0$, the following Jensen inequality holds:

$$\rho(\hat{\mathbb{E}}[X|\Omega_t]) \geq \hat{\mathbb{E}}[\rho(X)|\Omega_t].$$

Proof of Theorem 3.1. Let $\hat{X}_t^\varepsilon := X_t^\varepsilon - X_t^0$, $\hat{X}_0^\varepsilon := X_0^\varepsilon - X_0^0$, then

$$\begin{aligned}\hat{X}_t^\varepsilon = \hat{X}_0^\varepsilon &+ \int_0^t (b^\varepsilon(s, X_s^\varepsilon) - b^0(s, X_s^0))ds \\ &+ \sum_{i,j=1}^d \int_0^t (h_{ij}^\varepsilon(s, X_s^\varepsilon) - h_{ij}^0(s, X_s^0))d\langle B^i, B^j\rangle_s \\ &+ \sum_{j=1}^d \int_0^t (\sigma_j^\varepsilon(s, X_s^\varepsilon) - \sigma_j^0(s, X_s^0))dB_s^j,\end{aligned} \tag{2}$$

and

$$\begin{aligned}|\hat{X}_t^\varepsilon|^2 \leq C\{&|\hat{X}_0^\varepsilon|^2 + |\int_0^t (b^\varepsilon(s, X_s^\varepsilon) - b^\varepsilon(s, X_s^0))ds|^2 + |\int_0^t (b^\varepsilon(s, X_s^0) - b^0(s, X_s^0))ds|^2 \\ &+ \sum_{i,j=1}^d |\int_0^t (h_{ij}^\varepsilon(s, X_s^\varepsilon) - h_{ij}^\varepsilon(s, X_s^0))d\langle B^i, B^j\rangle_s|^2 \\ &+ \sum_{i,j=1}^d |\int_0^t (h_{ij}^\varepsilon(s, X_s^0) - h_{ij}^0(s, X_s^0))d\langle B^i, B^j\rangle_s|^2 \\ &+ \sum_{j=1}^d |\int_0^t (\sigma_j^\varepsilon(s, X_s^\varepsilon) - \sigma_j^\varepsilon(s, X_s^0))dB_s^j|^2 + \sum_{j=1}^d |\int_0^t (\sigma_j^\varepsilon(s, X_s^0) - \sigma_j^0(s, X_s^0))dB_s^j|^2\},\end{aligned} \tag{3}$$

taking the G-expectation on both sides of the above relation and from Proposition 2.3, we get

$$\begin{aligned}\hat{\mathbb{E}}[|\hat{X}_t^\varepsilon|^2] \leq C\{|\hat{X}_0^\varepsilon|^2 &+ \int_0^t \hat{\mathbb{E}}[|b^\varepsilon(s,X_s^\varepsilon) - b^\varepsilon(s,X_s^0)|^2]ds + \int_0^t \hat{\mathbb{E}}[|b^\varepsilon(s,X_s^0) - b^0(s,X_s^0)|^2]ds \\ &+ \sum_{i,j=1}^d \int_0^t \hat{\mathbb{E}}[|h_{ij}^\varepsilon(s,X_s^\varepsilon) - h_{ij}^\varepsilon(s,X_s^0)|^2]ds + \sum_{i,j=1}^d \int_0^t \hat{\mathbb{E}}[|h_{ij}^\varepsilon(s,X_s^0) - h_{ij}^0(s,X_s^0)|^2]ds \\ &+ \sum_{j=1}^d \int_0^t \hat{\mathbb{E}}[|\sigma_j^\varepsilon(s,X_s^\varepsilon) - \sigma_j^\varepsilon(s,X_s^0)|^2]ds + \sum_{j=1}^d \int_0^t \hat{\mathbb{E}}[|\sigma_j^\varepsilon(s,X_s^0) - \sigma_j^0(s,X_s^0)|^2]ds\},\end{aligned} \tag{4}$$

by Assumption 3.2, we have

$$\hat{\mathbb{E}}[|\hat{X}_t^\varepsilon|^2] \leq C^\varepsilon(T) + C_2 \int_0^t \alpha^2(s)\hat{\mathbb{E}}[\rho(|\hat{X}_s^\varepsilon|^2)]ds, \tag{5}$$

where

$$\begin{aligned}C^\varepsilon(t) := C\int_0^t \hat{\mathbb{E}}[|b^\varepsilon(s,X_s^0) - b^0(s,X_s^0)|^2]ds + C\sum_{i,j=1}^d \int_0^t \hat{\mathbb{E}}[|h_{ij}^\varepsilon(s,X_s^0) - h_{ij}^0(s,X_s^0)|^2]ds \\ + C\sum_{j=1}^d \int_0^t \hat{\mathbb{E}}[|\sigma_j^\varepsilon(s,X_s^0) - \sigma_j^0(s,X_s^0)|^2]ds + C|\hat{X}_0^\varepsilon|^2.\end{aligned}$$

Because ρ is concave and increasing, from Lemma 3.2, we have

$$\hat{\mathbb{E}}[|\hat{X}_t^\varepsilon|^2] \leq C^\varepsilon(T) + C_2 \int_0^t \alpha^2(s)\rho(\hat{\mathbb{E}}[|\hat{X}_s^\varepsilon|^2])ds. \tag{6}$$

Since as $\varepsilon \to 0$, $C^\varepsilon(T) \to 0$, hence, from Lemma 3.1, we get

$$\hat{\mathbb{E}}[|\hat{X}_t^\varepsilon|^2] \to 0, \text{ as } \varepsilon \to 0.$$

The proof is complete.

A special case of Assumption 3.2 is

Assumption 3.4 (Lipschitz condition). For any $x_1, x_2 \in R^n$, there exist constant $C_0 > 0$ such that

$$|\phi^\varepsilon(t,x_1) - \phi^\varepsilon(t,x_2)| \leq C_0|x_1 - x_2|, \quad t \in [0,T],$$

where $\phi = b, h_{ij}$ and σ_j, respectively, $i,j = 1,\cdots,d$.

Corollary 3.1. Under the Assumptions 3.1, 3.3 and 3.4, we have the convergence of the solution of the G-SDEs (3.2) in the sense of (3.3).

4 Stability theorem of G-backward stochastic differential equations

In this section, we give a stability theorem of backward stochastic differential equations driven by d-dimensional G-Brownian motion (G-BSDEs for short). Consider the following type of G-backward stochastic differential equations depending on a parameter ($\delta \geq 0$):

$$Y_t^\delta = \hat{\mathbb{E}}[\xi^\delta + \int_t^T f^\delta(s, Y_s^\delta)ds + \sum_{i,j=1}^{d} \int_t^T g_{ij}^\delta(s, Y_s^\delta)d\langle B^i, B^j\rangle_s|\Omega_t],\ t \in [0,T], \tag{1}$$

where $\xi^\delta \in L_G^1(\Omega_T; R^n)$ is given, and $f^\delta(\cdot, y), g_{ij}^\delta(\cdot, y) \in M_G^1(0,T; R^n)$.

We further make the following assumptions:

Assumption 4.1. For any $\delta \geq 0, y, y_1, y_2 \in R^n$,

(H1) $|f^\delta(t,y)| + \sum_{i,j=1}^{d} |g_{ij}^\delta(t,y)| \leq \beta(t) + C|y|$,

(H2)$|f^\delta(t,y_1) - f^\delta(t,y_2)| + \sum_{i,j=1}^{d} |g_{ij}^\delta(t,y_1) - g_{ij}^\delta(t,y_2)| \leq \rho(|y_1 - y_2|)$,

where $C > 0, \beta \in M_G^1([0,T]; R^+)$, and $\rho : (0,+\infty) \to (0,+\infty)$ is continuous, increasing, concave function satisfying $\rho(0+) = 0, \int_0^1 \frac{1}{\rho(r)}dr = +\infty$.

Assumption 4.2. (i) $\forall\ t \in [0,T]$, as $\delta \to 0$,

$$\int_t^T \hat{\mathbb{E}}[|\phi^\delta(s, Y_s^0) - \phi^0(s, Y_s^0)|]ds \to 0,$$

where $\phi = f, g_{ij}$ respectively, $i,j = 1, \cdots, d$.

(ii) As $\delta \to 0$,

$$\hat{\mathbb{E}}[|\xi^\delta - \xi^0|] \to 0.$$

Remark 4.1. Under the Assumptions 4.1 and 4.2, G-BSDEs (4.1) has a unique solution. The proof goes in a similar way as that in [2], and we omit it.

Theorem 4.1. Under the Assumptions 4.1 and 4.2, we have the following convergence: as $\delta \to 0$,

$$\forall\ t \in [0,T],\ \ \hat{\mathbb{E}}[|Y_t^\delta - Y_t^0|] \to 0. \tag{2}$$

Proof. Let $\hat{Y}_t^\delta := Y_t^\delta - Y_t^0, \hat{\xi}^\delta := \xi^\delta - \xi^0$, then

$$\begin{aligned}|\hat{Y}_t^\delta| &\le \hat{\mathbb{E}}[|\hat{\xi}^\delta| + \int_t^T |f^\delta(s,Y_s^\delta) - f^0(s,Y_s^0)|ds + \sum_{i,j=1}^d \int_t^T |g_{ij}^\delta(s,Y_s^\delta) - g_{ij}^0(s,Y_s^0)|d\langle B^i, B^j\rangle_s|\Omega_t] \\ &\le \hat{\mathbb{E}}[|\hat{\xi}^\delta| + \int_t^T |f^\delta(s,Y_s^0) - f^0(s,Y_s^0)|ds + \sum_{i,j=1}^d \int_t^T |g_{ij}^\delta(s,Y_s^0) - g_{ij}^0(s,Y_s^0)|d\langle B^i, B^j\rangle_s \\ &+ \int_t^T |f^\delta(s,Y_s^\delta) - f^\delta(s,Y_s^0)|ds + \sum_{i,j=1}^d \int_t^T |g_{ij}^\delta(s,Y_s^\delta) - g_{ij}^\delta(s,Y_s^0)|d\langle B^i, B^j\rangle_s|\Omega_t].\end{aligned} \tag{3}$$

Taking the G-expectation on both sides of (3), we have

$$\begin{aligned}\hat{\mathbb{E}}[|\hat{Y}_t^\delta|] &\le \hat{\mathbb{E}}[|\hat{\xi}^\delta|] + \int_t^T \hat{\mathbb{E}}[|f^\delta(s,Y_s^\delta) - f^0(s,Y_s^0)|]ds + C\sum_{i,j=1}^d \int_t^T \hat{\mathbb{E}}[|g_{ij}^\delta(s,Y_s^\delta) - g_{ij}^0(s,Y_s^0)|]ds \\ &+ \int_t^T \hat{\mathbb{E}}[|f^\delta(s,Y_s^\delta) - f^\delta(s,Y_s^0)|]ds + C\sum_{i,j=1}^d \int_t^T \hat{\mathbb{E}}[|g_{ij}^\delta(s,Y_s^\delta) - g_{ij}^\delta(s,Y_s^0)|]ds.\end{aligned} \tag{4}$$

From the Assumption 4.1, Propositions 2.1 and 2.3 as well as Lemma 3.2, we have

$$\begin{aligned}\hat{\mathbb{E}}[|\hat{Y}_t^\delta|] &\le C^\delta(0) + K_1\int_t^T \hat{\mathbb{E}}[\rho(|\hat{Y}_s^\delta|)]ds \\ &\le C^\delta(0) + K_1\int_t^T \rho(\hat{\mathbb{E}}[|\hat{Y}_s^\delta|])ds.\end{aligned} \tag{5}$$

where

$$C^\delta(0) := \hat{\mathbb{E}}[|\hat{\xi}^\delta|] + \int_0^T \hat{\mathbb{E}}[|f^\delta(s,Y_s^0) - f^0(s,Y_s^0)|]ds + C\sum_{i,j=1}^d \int_0^T \hat{\mathbb{E}}[|g_{ij}^\delta(s,Y_s^0) - g_{ij}^0(s,Y_s^0)|]ds.$$

Since as $\delta \to 0$, $C^\delta(0) \to 0$, hence, from Lemma 3.1, we have

$$\hat{\mathbb{E}}[|\hat{Y}_t^\delta|] \to 0.$$

The proof is complete.

A special case of Assumption 4.1 is

Assumption 4.3. For any $\delta \ge 0, y_1, y_2 \in R^n$, there exist constant $C_0 > 0$ such that

$$|\phi^\delta(t,y_1) - \phi^\delta(t,y_2)| \le C_0|y_1 - y_2|, \ t \in [0,T],$$

$\phi = f, g_{ij}$ respectively, $i,j = 1,\cdots,d$.

Corollary 4.1. Under the Assumptions 4.2 and 4.3, we have the convergence of the solution of the G-BSDEs (4.1) in the sense of (4.2).

5 Forward-backward stochastic differential equations

The goal of this section is to show the existence and uniqueness of forward-backward stochastic differential equations driven by G-Brownian motion. For notational simplification, we only consider the case of 1-dimensional G-Brownian motion. However, our method can be easily extend to the case of multi-dimensional G-Brownian motion. We consider the following system:

$$\begin{cases} X_t = x + \int_0^t b(s, X_s, Y_s)ds + \int_0^t h(s, X_s, Y_s)d\langle B\rangle_s + \int_0^t \sigma(s, X_s, Y_s)dB_s, \\ Y_t = \hat{\mathbb{E}}[\xi + \int_t^T f(s, X_s, Y_s)ds + \int_t^T g(s, X_s, Y_s)d\langle B\rangle_s|\Omega_t], \ t \in [0, T], \end{cases} \tag{1}$$

where the initial condition $x \in R$, the terminal data $\xi \in L_G^2(\Omega_T; R)$, and b, h, σ, f, g are given functions satisfying $b(\cdot, x, y), h(\cdot, x, y), \sigma(\cdot, x, y), f(\cdot, x, y), g(\cdot, x, y) \in M_G^2([0, T]; R)$ for any $(x, y) \in R^2$ and the Lipschitz condition, i.e., $|\phi(t, x, y) - \phi(t, x', y')| \leq K(|x - x'| + |y - y'|)$, for each $t \in [0, T], (x, y) \in R^2, (x', y') \in R^2, \phi = b, h, \sigma, f$ and g, respectively. The solution is a pair of processes $(X, Y) \in M_G^2(0, T; R) \times M_G^2(0, T; R)$.

This model is called forward-backward because the two components in the system (1) are solutions, respectively, of a G-forward and a G-backward stochastic differential equation.

We first introduce the following mappings on a fixed interval $[0, T]$:

$$\Lambda_\cdot^i : M_G^2(0, T; R) \times M_G^2(0, T; R) \to M_G^2(0, T; R) \times M_G^2(0, T; R), i = 1, 2,$$

by setting $\Lambda_t^i, i = 1, 2, t \in [0, T]$, with

$$\begin{aligned} \Lambda_t^1(X, Y) &= x + \int_0^t b(s, X_s, Y_s)ds + \int_0^t h(s, X_s, Y_s)d\langle B\rangle_s + \int_0^t \sigma(s, X_s, Y_s)dB_s, \\ \Lambda_t^2(X, Y) &= \hat{\mathbb{E}}[\xi + \int_t^T f(s, X_s, Y_s)ds + \int_t^T g(s, X_s, Y_s)d\langle B\rangle_s|\Omega_t]. \end{aligned} \tag{2}$$

Lemma 5.1. For any $(X, Y), (X', Y') \in M_G^2(0, T; R) \times M_G^2(0, T; R)$, we have the following estimates:

$$\begin{aligned} \hat{\mathbb{E}}[|\Lambda_t^1(X, Y) - \Lambda_t^1(X', Y')|^2] &\leq C \int_0^t \hat{\mathbb{E}}[|X_s - X'_s|^2 + |Y_s - Y'_s|^2]ds, t \in [0, T], \\ \hat{\mathbb{E}}[|\Lambda_t^2(X, Y) - \Lambda_t^2(X', Y')|^2] &\leq C' \int_t^T \hat{\mathbb{E}}[|X_s - X'_s|^2 + |Y_s - Y'_s|^2]ds, t \in [0, T], \end{aligned} \tag{3}$$

where $C = 24K^2, C' = 8K^2, K$ is Lipschitz coefficient.
Proof.

$$\begin{aligned}\hat{\mathbb{E}}[|\Lambda_t^1(X,Y) - \Lambda_t^1(X',Y')|^2] &\leq 4\int_0^t \hat{\mathbb{E}}[|b(s,X_s,Y_s) - b(s,X_s',Y_s')|^2]ds \\ &+ 4\int_0^t \hat{\mathbb{E}}[|h(s,X_s,Y_s) - h(s,X_s',Y_s')|^2]ds \\ &+ 4\int_0^t \hat{\mathbb{E}}[|\sigma(s,X_s,Y_s) - \sigma(s,X_s',Y_s')|^2]ds \\ &\leq 24K^2\int_0^t \hat{\mathbb{E}}[|X_s - X_s'|^2 + |Y_s - Y_s'|^2]ds.\end{aligned}$$

And since

$$\begin{aligned}|\Lambda_t^2(X,Y) - \Lambda_t^2(X',Y')|^2 \leq 2\hat{\mathbb{E}}[|\int_t^T f(s,X_s,Y_s) - f(s,X_s',Y_s')ds|^2 \\ + |\int_t^T g(s,X_s,Y_s) - g(s,X_s',Y_s')ds|^2|\Omega_t],\end{aligned}$$

then

$$\begin{aligned}\hat{\mathbb{E}}[|\Lambda_t^2(X,Y) - \Lambda_t^2(X',Y')|^2] \leq 2\hat{\mathbb{E}}[|\int_t^T f(s,X_s,Y_s) - f(s,X_s',Y_s')ds|^2 \\ + |\int_t^T g(s,X_s,Y_s) - g(s,X_s',Y_s')ds|^2] \\ \leq 2\int_t^T \hat{\mathbb{E}}[|f(s,X_s,Y_s) - f(s,X_s',Y_s')|^2]ds \\ + 2\int_t^T \hat{\mathbb{E}}[|g(s,X_s,Y_s) - g(s,X_s',Y_s')|^2]ds \\ \leq 8K^2\int_t^T \hat{\mathbb{E}}[|X_s - X_s'|^2 + |Y_s - Y_s'|^2]ds.\end{aligned}$$

Let us consider the space $M_G^2(0,T;R)\times M_G^2(0,T;R)$, with the norm $||(X,Y)||_{M_G^2(0,T)\times M_G^2(0,T)} := ||X||_{M_G^2(0,T)} + ||Y||_{M_G^2(0,T)} = \int_0^T \hat{\mathbb{E}}[|X_s|^2]ds + \int_0^T \hat{\mathbb{E}}[|Y_s|^2]ds$, this is a Banach space.

Theorem 5.1. Let time T satisfy $(2\sqrt{6} + 2\sqrt{2})K\sqrt{T} < 1$, then there exists a unique solution $(X,Y) \in M_G^2(0,T;R) \times M_G^2(0,T;R)$ of the forward-backward stochastic differential equation (1).

Proof. Let us consider the space $M_G^2(0,T;R) \times M_G^2(0,T;R)$, with the norm

$$||(X,Y)||_{M_G^2(0,T)\times M_G^2(0,T)} := ||X||_{M_G^2(0,T)} + ||Y||_{M_G^2(0,T)}.$$

We can view the system (1) as the operator $\Lambda_t(X,Y) := \begin{pmatrix} \Lambda_t^1(X,Y) \\ \Lambda_t^2(X,Y) \end{pmatrix}$, thus

$$\begin{aligned}
&||\Lambda_t(X,Y) - \Lambda_t(X',Y')||_{M_G^2(0,T)\times M_G^2(0,T)} \\
&= ||\Lambda_t^1(X,Y) - \Lambda_t^1(X',Y')||_{M_G^2(0,T)} + ||\Lambda_t^2(X,Y) - \Lambda_t^2(X',Y')||_{M_G^2(0,T)} \\
&= (\int_0^T \hat{\mathbb{E}}[(\Lambda_t^1(X,Y) - \Lambda_t^1(X',Y'))^2]dt)^{\frac{1}{2}} \\
&+ (\int_0^T \hat{\mathbb{E}}[(\Lambda_t^2(X,Y) - \Lambda_t^2(X',Y'))^2]dt)^{\frac{1}{2}}.
\end{aligned} \tag{4}$$

Because the terminal data $\xi \in L_G^2(\Omega_T; R)$, and b, h, σ, f, g are given functions satisfying $b(\cdot,x,y), h(\cdot,x,y),\ \sigma(\cdot,x,y), f(\cdot,x,y), g(\cdot,x,y) \in M_G^2([0,T];R)$ for any $(x,y) \in R^2$ and the Lipschitz condition, we can prove $||\Lambda_t(X,Y)||_{M_G^2(0,T)\times M_G^2(0,T)} < +\infty, \forall (X,Y) \in M_G^2(0,T;R) \times M_G^2(0,T;R)$. Next, we prove it is a contraction mapping. From the Lemma 5.1, we can obtain

$$\begin{aligned}
&||\Lambda_t(X,Y) - \Lambda_t(X',Y')||_{M_G^2(0,T)\times M_G^2(0,T)} \\
&\leq (\int_0^T C \int_0^t \hat{\mathbb{E}}[|X_s - X_s'|^2 + |Y_s - Y_s'|^2]dsdt)^{\frac{1}{2}} \\
&+ (\int_0^T C' \int_t^T \hat{\mathbb{E}}[|X_s - X_s'|^2 + |Y_s - Y_s'|^2]dsdt)^{\frac{1}{2}} \\
&\leq (\sqrt{C} + \sqrt{C'})\sqrt{T} \int_0^T \hat{\mathbb{E}}[|X_s - X_s'|^2 + |Y_s - Y_s'|^2]ds \\
&= (\sqrt{C} + \sqrt{C'})\sqrt{T}||(X - X', Y - Y')||_{M_G^2(0,T)\times M_G^2(0,T)}.
\end{aligned} \tag{5}$$

From the assumption $(2\sqrt{6} + 2\sqrt{2})K\sqrt{T} < 1$, we can obtain that $\Lambda_t(X,Y)$ is a contraction mapping. Hence a unique fixed point for Λ exists and this is the solution of our system (1). The proof is complete.

In the last section, we present stability theorem for forward-backward stochastic differential equations driven by G-Brownian motion.

6 Stability theorem of forward-backward stochastic differential equations

Consider a family of forward-backward stochastic differential equations with parameter $(\gamma \geq 0)$,

$$\begin{cases} X_t^\gamma = x^\gamma + \int_0^t b^\gamma(s, X_s^\gamma, Y_s^\gamma)ds + \int_0^t h^\gamma(s, X_s^\gamma, Y_s^\gamma)d\langle B\rangle_s + \int_0^t \sigma^\gamma(s, X_s^\gamma, Y_s^\gamma)dB_s, \\ Y_t^\gamma = \hat{\mathbb{E}}[\xi^\gamma + \int_t^T f^\gamma(s, X_s^\gamma, Y_s^\gamma)ds + \int_t^T g^\gamma(s, X_s^\gamma, Y_s^\gamma)d\langle B\rangle_s|\Omega_t], \ t \in [0,T], \end{cases} \tag{1}$$

where the initial condition $x^\gamma \in R$, the terminal data $\xi^\gamma \in L_G^2(\Omega_T; R)$, and $b^\gamma, h^\gamma, \sigma^\gamma, f^\gamma, g^\gamma$ are given functions satisfying $b^\gamma(\cdot,x,y), h^\gamma(\cdot,x,y), \sigma^\gamma(\cdot,x,y), f^\gamma(\cdot,x,y), g^\gamma(\cdot,x,y) \in M_G^2([0,T]; R)$ for any $(x,y) \in R^2$ and the Lipschitz condition, i.e.,
Assumption 6.1.

$$|\phi(t,x,y) - \phi(t,x',y')| \leq K(|x-x'| + |y-y'|),$$

for each $t \in [0,T], (x,y) \in R^2, (x',y') \in R^2, \phi = b^\gamma, h^\gamma, \sigma^\gamma, f^\gamma$ and g^γ, respectively.

We further make the following assumption:
Assumption 6.2. (i) $\forall\ t \in [0,T]$, as $\gamma \to 0$,

$$\int_0^t \hat{\mathbb{E}}[|\phi^\gamma(s, X_s^0, Y_s^0) - \phi^0(s, X_s^0, Y_s^0)|^2]ds \to 0,$$

where $\phi = b, h, \sigma, f$ and g, respectively.
(ii) As $\gamma \to 0$,

$$x^\gamma \to x^0, \ \ \hat{\mathbb{E}}[|\xi^\gamma - \xi^0|^2] \to 0.$$

Theorem 6.1. Under the Assumptions 6.1 and 6.2, then as $\gamma \to 0$, (X_t^γ, Y_t^γ) convergence to (X_t^0, Y_t^0) in the sense that

$$\forall\ t \in [0,T], \ \ \hat{\mathbb{E}}[|X_t^\gamma - X_t^0|^2 + |Y_t^\gamma - Y_t^0|^2] \to 0. \tag{2}$$

The proof of Theorem 6.1 is similar to that of the Theorem 3.1, we omit it. **Acknowledgment.** This work has been supported by The Natural Basic Research Program of China (973 Program)(Grant No. 2007CB814901) and The Scientific Research Foundation of Yunnan Province Education Committee (2011C120). The authors thank the referee for their helpful suggestions which essentially improved the presentation of this paper.

References

[1] F. Antonelli, *Backward-forward stochastic differential equations*, Ann. Appl. Probab. **3** (1993), 777-793.

[2] X. Bai and Y. Lin, *On the existence and uniqueness of solutions to stochastic differential equations driven by G-Brownian motion with integral-Lipschitz coefficients*, arXiv: math. PR/ 1002.1046V2, 2010.

[3] J. M. Bismut, *Conjugate convex functions in optimal stochastic control*, J. Math. Anal. Apl. **44** (1973), 384-404.

[4] J. Y. Chemin and N. Lerner, *Flow of Non-Lipschitz Vector-Fields and Navier-Stokes Equations*, J. Differential Equations **121** (1995), 314-328.

[5] L. Denis, M. Hu and S. Peng, *Function spaces and capacity related to a sublinear expectation: application to G-Brownian motion paths*, Potential Anal., 2010, doi.10.1007/s11118-010-9185-x.

[6] N. El Karoui, S. Peng and M. C. Quenez, *Backward stochastic differential equation in finance*, Mathematical Finance **7** (1997), 1-71.

[7] F. Gao, *Pathwise properties and homeomorphic flows for stochastic differential equations driven by G-Brownian motion*, Stoch. Proc. Appl. **119** (2009), 3356-3382.

[8] F. Gao and H. Jiang, *Large deviations for stochastic differential equations driven by G-Brownian motion*, Stoch. Proc. Appl. **120** (2010), 2212-2240.

[9] Y. Hu and S. Peng, *Solution of forward-backward stochastic differential equations*, Probab. Theory Relat. Fields **103** (1995), 273-283.

[10] Y. Hu and S. Peng, *A stability theorem of backward stochastic differential equations and its application*, Comptes Rendus de l'Académie des Sciences-Series I-Mathematics **324** (1997), 1059-1064.

[11] J. Ma, P. Protter and J. Yong, *Solving forward-backward stochastic differential equations explicitly - a four step scheme*, Probab. Theory Relat. Fields **98** (1994), 339-359.

[12] E. Pardoux and S. Peng, *Adapted solution of a backward stochastic differential equation*, Systems Control Letters **14** (1990), 61-74.

[13] S. Peng, *G-Expectation, G-Brownian motion and related stochasticcal culus of Itô's type*, in: Benth, etal. (Eds.), Proceedings of the 2005 Abel Symposium 2, Springer-Verlag, 2006, pp. 541-567.

[14] S. Peng, *Multi-dimensional G-Brownian motion and related stochastic calculus under G-expectation*, Stoch. Proc. Appl. **118** (2008), 2223-2253.

[15] S. Peng, *Nonlinear Expectations and Stochastic Calculus under Uncertainty-with Robust Central Limit Theorem and G-Brownian Motion*, arXiv:math. PR/1002.4546v1, 2010.

[16] P. Protter, *Stochastic integration and differential equations - a new approach*, Springer, 1990.

[17] H. M. Soner, N. Touzi and J. Zhang, *Martingale representation theorem for the G-expectation*, arXiv:math. PR/1001.3802v2, 2010.

[18] Y. Song, *Some properties on G-evalution and its applications to G-martingale decomposition*, arXiv:math. PR/1001.2802v2, 2010.

[19] J. Xu and B. Zhang, *Martingale characterization of G-Brownian motion*, Stoch. Proc. Appl. **119** (2009), 232-248.

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