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PROPERTIES OF THE NEAREST PARAMETRIC FORM APPROXIMATION OPERATOR OF FUZZY NUMBERS

Majid Amirfakhrian

Abstract

In many applications of fuzzy logic and fuzzy mathematics we need (or it is better) to work with the same fuzzy numbers. In this work we present some properties of parametric m-degree polynomial approximation operator of fuzzy numbers.

1 Introduction

In some applications of fuzzy logic, we need to compare two fuzzy numbers. For this purpose we find two quantities related to the fuzzy numbers to make them comparable.

There are many literatures which authors tried to approximate a fuzzy number by a simpler one [1, 2, 3, 8, 9, 10, 14, 15, 16, 20]. Also there are some distances defined by authors to compare fuzzy numbers [18, 19].

Obviously, if we use a defuzzification rule which replaces a fuzzy set by a single number, we generally loose too many important information. Also, an interval approximation is considered for fuzzy numbers in [9], where a fuzzy computation problem is converted into interval arithmetic problem. But, in this case, we loose the fuzzy central concept. Even in some works such as [3, 15, 16, 20], authors solve an optimization problem to obtain the nearest triangular or trapezoidal fuzzy number which is related to an arbitrary fuzzy number, however in these cases there is not any guarantee to have the same

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modal value (or interval). But by parametric polynomial approximation we are able to approximate many fuzzy numbers in a good manner.

Recently the problem of finding the nearest parametric approximation of a fuzzy number with respect to the average Euclidean distance is completely solved in [4]. A. I. Ban pointed out the wrongs and inadvertences in some recent papers, then correct the results in [5]. A parametric fuzzy approximation method based on the decision makers strategy as an extension of trapezoidal approximation of a fuzzy number offered in [17]. An improvement of the nearest trapezoidal approximation operator preserving the expected interval, which is proposed by Grzegorzewski and Mrowka is studied in [21]. There are some trapezoidal approximation operators introduced in [12, 13, 22]

The structure of the present paper is as follows. In Section 2 we introduce the basic concepts of our work. In Section 3 we represent an $m-$source distance and some useful lemmas and theorems, and in Section 4 some properties of this distance are checked.

2 Preliminaries

Let $F(\mathbb{R})$ be the set of all normal and convex fuzzy numbers on the real line [23]. A fuzzy number with LR form introduced in [7].

Definition 2.1. [1] A generalized LR fuzzy number $\tilde{A}$ with the membership function $\mu_{\tilde{A}}(x), x \in \mathbb{R}$ can be defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} l_{\tilde{A}}(x), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ r_{\tilde{A}}(x), & c \leq x \leq d, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where $l_{\tilde{A}}(x)$ is the left membership function, that is an increasing function on $[a, b]$, and $r_{\tilde{A}}(x)$ is the right membership function, that is a decreasing function on $[c, d]$, such that $l_{\tilde{A}}(a) = r_{\tilde{A}}(d) = 0$ and $l_{\tilde{A}}(b) = r_{\tilde{A}}(c) = 1$. In addition, if $l_{\tilde{A}}(x)$ and $r_{\tilde{A}}(x)$ are linear, then $\tilde{A}$ is a trapezoidal fuzzy number which is denoted by (a, b, c, d). If $b = c$, we denoted it by (a, c, d), which is a triangular fuzzy number.

α-cut of a fuzzy number $\tilde{a}$ is defined by [7],

$$[\tilde{a}]^\alpha = \begin{cases} \{t \in \mathbb{R} \mid \mu_{\tilde{a}}(t) \geq \alpha\} & , \quad \alpha > 0, \\ \overline{\{t \in \mathbb{R} \mid \mu_{\tilde{a}}(t) > \alpha\}} & , \quad \alpha = 0. \end{cases} \tag{2}$$

Definition 2.2. [18] A continuous function $s : [0,1] \longrightarrow [0,1]$ with the following properties is a regular reducing function :

1. $s(r)$ is increasing;
2. $s(0) = 0$;
3. $s(1) = 1$;
4. $\int_0^1 s(r)dr = \frac{1}{2}$.

The *parametric form* of a fuzzy number is shown by $\tilde{v} = (\underline{v}(r), \overline{v}(r))$, where the functions $\underline{v}(r)$ and $\overline{v}(r)$, $0 \leq r \leq 1$, satisfy the following requirements:

1. $\underline{v}(r)$ is a monotonically increasing left continuous function.
2. $\overline{v}(r)$ is a monotonically decreasing left continuous function.
3. $\underline{v}(r) \leq \overline{v}(r) \quad , \quad 0 \leq r \leq 1$.

Definition 2.3. [18] The *value* and *ambiguity* of a fuzzy number $\tilde{v}$ is defined by the following relations,

$$\begin{array}{l} Val(\tilde{v}) = \int_0^1 s(r)[\overline{v}(r) + \underline{v}(r)]dr, \\ Amb(\tilde{v}) = \int_0^1 s(r)[\overline{v}(r) - \underline{v}(r)]dr. \end{array} \tag{3}$$

Definition 2.4. We say a fuzzy number $\tilde{v}$ has an $m-$degree polynomial form if there exist two polynomials $p_m(r)$ and $q_m(r)$, of degree at most m; such that $\tilde{v} = (p_m(r), q_m(r))$.

Let $PF_m(\mathbb{R})$ be the set of all $m-$degree polynomial form fuzzy numbers.

Definition 2.5. [1] For $\tilde{u}, \tilde{v} \in F(\mathbb{R})$, we define ***source distance*** of $\tilde{u}$ and $\tilde{v}$ by

$$D(\tilde{u}, \tilde{v}) = \frac{1}{2}\big\{ |Val(\tilde{u}) - Val(\tilde{v})| + |Amb(\tilde{u}) - Amb(\tilde{v})| + d_H([\tilde{u}]^1, [\tilde{v}]^1)\big\}, \tag{4}$$

where d_H is the Hausdorff metric.

The source distance, D, is a metric on the set of all trapezoidal fuzzy numbers and a pseudo-metric on $F(\mathbb{R})$.

Definition 2.6. [1] Let $AF(\mathbb{R})$ be a subset of $F(\mathbb{R})$. $\tilde{v}^* \in AF(\mathbb{R})$, is a near approximation of an arbitrary fuzzy number $\tilde{u} \in F(\mathbb{R})$ if and only if

$$D(\tilde{v}^*, \tilde{u}) = \min_{\tilde{v} \in AF(\mathbb{R})} D(\tilde{v}, \tilde{u}). \tag{5}$$

Definition 2.7. [1] Let $\overline{u}, \underline{u} \in C^k[0,1]$, for positive integer $k > 0$. We define *k-validity* and *k-unworthiness* of a fuzzy number $\tilde{u}$ by the following relations,

$$\begin{array}{l} V(\tilde{u}^{(k)}) = \int_0^1 s(r)[\overline{u}^{(k)}(r) + \underline{u}^{(k)}(r)]dr, \\ A(\tilde{u}^{(k)}) = \int_0^1 s(r)[\overline{u}^{(k)}(r) - \underline{u}^{(k)}(r)]dr. \end{array} \tag{6}$$

0-validity and 0-unworthiness of a fuzzy number are value and ambiguity of it, respectively.

Definition 2.8. [1], $\tilde{v}^* \in PF_m(\mathbb{R})$, is the nearest approximation of an arbitrary fuzzy number $\tilde{u} \in F(\mathbb{R})$ out of $PF_m(\mathbb{R})$, if and only if

1. $\tilde{v}^*$ is a near approximation of $\tilde{u}$.
2. If $m \geq 2$, then

$$D^*_m(\tilde{v}^*, \tilde{u}) = \min_{\tilde{v} \in PF_m(\mathbb{R})} D^*_m(\tilde{v}, \tilde{u}), \tag{7}$$

   where

$$D^*_m(\tilde{v}, \tilde{u}) = \sum_{k=1}^{m-1} \{|V(\tilde{v}^{(k)}) - V(\tilde{u}^{(k)})| + |A(\tilde{v}^{(k)}) - A(\tilde{u}^{(k)})|\}. \tag{8}$$

Theorem 2.1. *Let $m \geq 2$ and $\tilde{v} \in PF_m(\mathbb{R})$ is a near approximation of fuzzy number $\tilde{u}$. $\tilde{v}$ is the nearest approximation of $\tilde{u}$ out of $PF_m(\mathbb{R})$, if and only if for $k = 1, \ldots, m-1$, $\tilde{v}$ and $\tilde{u}$ have the same k-validity and k-unworthiness.*

Proof. See [1] □

Lemma 2.2. *Let $\tilde{u}$ be a generalized LR fuzzy number. If for positive integer m we have $\underline{u}, \overline{u} \in C^{m-1}[0,1]$, then the nearest approximation of $\tilde{u}$ out of $PF_m(\mathbb{R})$ exists.*

For a nonnegative integer j, we define j^{th}*-source number* by

$$I_j = \int_0^1 r^j s(r)dr. \tag{9}$$

Lemma 2.3. *Let I_j be the j^{th}-source number, then $0 \leq \ldots < I_2 < I_1 < I_0 = \frac{1}{2}$.*

Proof. By using the definition of a j^{th}-source number and a regular reducing function, the proof is straightforward. □

We denote the set of all the nearest approximations of a fuzzy number $\tilde{u}$, from PF_m, by $N^*_{PF_m}(\tilde{u})$.

3 m-Source Distance

Definition 3.1. For $\tilde{u}, \tilde{v} \in F(\mathbb{R})$, we define m-***source distance*** of $\tilde{u}$ and $\tilde{v}$ by

$$D_m(\tilde{u}, \tilde{v}) = D(\tilde{u}, \tilde{v}) + D^*_m(\tilde{u}, \tilde{v}).$$

where D is source distance and D^* defined in (8).

Theorem 3.1. *For $\tilde{u}, \tilde{v}, \tilde{w} \in F(\mathbb{R})$ the distance, D_m, satisfies the following properties:*

1. $D_m(\tilde{u}, \tilde{u}) = 0,$
2. $D_m(\tilde{u}, \tilde{v}) = D_m(\tilde{v}, \tilde{u}),$
3. $D_m(\tilde{u}, \tilde{w}) \leq D_m(\tilde{u}, \tilde{v}) + D_m(\tilde{v}, \tilde{w}),$
4. $D_m(k\tilde{u}, k\tilde{v}) = |k| D_m(\tilde{u}, \tilde{v})$ *for* $k \in \mathbb{R},$
5. $D_m(\tilde{u} + \tilde{w}, \tilde{v} + \tilde{z}) \leq D_m(\tilde{u}, \tilde{v}) + D_m(\tilde{w}, \tilde{z}).$

Example 3.1. *Let* $\mu_{\tilde{u}}(x) = \chi_{\{a\}}(x)$ *and* $\mu_{\tilde{v}}(x) = \chi_{\{b\}}(x)$*, then*

$$D_m(\tilde{u}, \tilde{v}) = |a - b|.$$

Proposition 3.2. *The fuzzy number $\tilde{v}^*$ is a nearest approximation of $\tilde{u}$ out of $PF_m(\mathbb{R})$ if and only if*

$$D_m(\tilde{v}^*, \tilde{u}) = \min_{\tilde{v} \in PF_m(\mathbb{R})} D_m(\tilde{v}, \tilde{u}). \tag{10}$$

Theorem 3.3. *Let $\tilde{u}$ be a fuzzy number. If for a positive integer m we have $\underline{u}, \overline{u} \in C^{m-1}[0,1]$, then $\tilde{v}$ is the nearest approximation of $\tilde{u}$ out of $PF_m(\mathbb{R})$ if and only if $D_m(\tilde{v}, \tilde{u}) = 0$.*

Theorem 3.4. *The nearest approximation of an m-degree polynomial form fuzzy number, out of $PF_m(\mathbb{R})$, is itself.*

Proof. Let $\tilde{v} \in PF_m(\mathbb{R})$ be the nearest approximation of , $\tilde{u} \in PF_m(\mathbb{R})$. Also let $\underline{v}(r) = \underline{d}_m r^m + \ldots + \underline{d}_1 r + \underline{d}_0$ and $\underline{u}(r) = \underline{c}_m r^m + \ldots + \underline{c}_1 r + \underline{c}_0$.

Let $w(r) = \sum_{j=0}^{m} a_j r^j$, where $a_j = \underline{c}_j - \underline{d}_j$ for $j = 0, 1, \ldots, m$. Thus $w(1) = 0$ and

$$\int_0^1 w^{(k)}(r) s(r) dr = 0 \qquad , \qquad k = 0, 1, \ldots, m-1.$$

Therefore we have a homogeneous system of linear equations with the following nonsingular matrix coefficients

$$\begin{pmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ I_0 & I_1 & I_2 & I_3 & \cdots & I_m \\ 0 & I_0 & 2I_1 & 3I_2 & \cdots & mI_{m-1} \\ 0 & 0 & 2I_0 & 3!I_1 & \cdots & m(m-1)I_{m-2} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & m!I_1. \end{pmatrix}$$

Therefore $w(r) \equiv 0$. i.e. $\underline{u}(r) \equiv \underline{v}(r)$. In a similar way, $\overline{u}(r) \equiv \overline{v}(r)$. Thus $\tilde{u} = \tilde{v}$. □

Corollary 3.5. *$D_m(.,.)$ is a metric on $PF_m(\mathbb{R})$.*

Lemma 3.6. *Let $\tilde{u}$ be a fuzzy number. For all $m \geq 1$ if $N^*_{PF_m}(\tilde{u})$ is not empty, then we have*

$$|N^*_{PF_m}(\tilde{u})| = 1.$$

Proof. Let $\tilde{u}^*_1$ and $\tilde{u}^*_2$ be the nearest approximations of $\tilde{u}$ out of $PF_m(\mathbb{R})$. $D_m(\tilde{u}^*_1, \tilde{u}) = D_m(\tilde{u}^*_1, \tilde{u}) = 0$.

$$D_m(\tilde{u}^*_1, \tilde{u}^*_2) \leq D_m(\tilde{u}^*_1, \tilde{u}) + D_m(\tilde{u}, \tilde{u}^*_2) = 0.$$

From Lemma 3.4 we have $\tilde{u}^*_1 = \tilde{u}^*_2$. □

Lemma 3.7. *Let $\tilde{u}^*$ and $\tilde{v}^*$ be the nearest approximations of two fuzzy numbers $\tilde{u}$ and $\tilde{v}$, respectively. Then we have*

$$D_m(\tilde{u}^*, \tilde{v}^*) = D_m(\tilde{u}, \tilde{v})$$

Proof.

$$D_m(\tilde{u}^*, \tilde{v}^*) \leq D_m(\tilde{u}^*, \tilde{u}) + D_m(\tilde{u}, \tilde{v}) + D_m(\tilde{v}, \tilde{v}^*) = D_m(\tilde{u}, \tilde{v}).$$

In a similar way $D_m(\tilde{u}, \tilde{v}) \leq D_m(\tilde{u}^*, \tilde{v}^*)$. □

Corollary 3.8. *If $\tilde{u}$ be an l-degree polynomial form fuzzy number, where $l \leq m$, then $\tilde{v}^* = \tilde{u}$.*

Lemma 3.9. *Let $\tilde{v}$ and $\tilde{u}$ be two fuzzy numbers, where $\underline{u}, \underline{v}, \overline{u}, \overline{v} \in C^{m-1}[0,1]$. If $D_m(\tilde{u}, \tilde{v}) = 0$, then there are two sequences of points $\{\delta_{i,k}\}_{k=0}^{m-1}$, $i = 1, 2$ such that for $k = 0, 1, \cdots, m-1$,*

$$\underline{u}^{(k)}(\delta_{1,k}) = \underline{v}^{(k)}(\delta_{1,k}),$$

and

$$\overline{u}^{(k)}(\delta_{2,k}) = \overline{v}^{(k)}(\delta_{2,k}).$$

Proof. Let for two fuzzy numbers $\tilde{v}$ and $\tilde{u}$ we have $D_m(\tilde{u},\tilde{v}) = 0$. Thus

$$\begin{array}{lcl} [\tilde{u}]^1 = [\tilde{v}]^1, & & \\ Val(\tilde{u}) = Val(\tilde{v}), & & \\ Amb(\tilde{u}) = Amb(\tilde{v}), & & \\ V(\tilde{v}^{(k)}) = V(\tilde{u}^{(k)}) & , & k = 1,\ldots,m-1\ , \\ A(\tilde{v}^{(k)}) = A(\tilde{u}^{(k)}) & , & k = 1,\ldots,m-1\ , \end{array}$$

so

$$\begin{array}{l} \int_0^1 s(r)[\overline{u}^{(k)}(r) + \underline{u}^{(k)}(r)]dr = \int_0^1 s(r)[\overline{v}^{(k)}(r) + \underline{v}^{(k)}(r)]dr, \\ \int_0^1 s(r)[\overline{u}^{(k)}(r) - \underline{u}^{(k)}(r)]dr = \int_0^1 s(r)[\overline{v}^{(k)}(r) - \underline{v}^{(k)}(r)]dr. \end{array}$$

Therefore

$$\int_0^1 s(r)[\overline{u}^{(k)}(r) - \overline{v}^{(k)}(r)]dr = \pm \int_0^1 s(r)[\underline{u}^{(k)}(r) - \underline{v}^{(k)}(r)]dr,$$

Thus

$$\begin{array}{lcl} \int_0^1 s(r)[\overline{u}^{(k)}(r) - \overline{v}^{(k)}(r)]dr = 0 & , & k = 1,\ldots,m-1\ , \\ \int_0^1 s(r)[\underline{u}^{(k)}(r) - \underline{v}^{(k)}(r)]dr = 0 & , & k = 1,\ldots,m-1\ , \end{array}$$

Thus by mean value theorem for integrals, for any $k = 0,1,\ldots,m-1$, there are two numbers $\delta_{1,k}$ and $\delta_{2,k}$ such that

$$\begin{array}{l} \overline{u}^{(k)}(\delta_{1,k}) = \overline{v}^{(k)}(\delta_{1,k}), \\ \underline{u}^{(k)}(\delta_{2,k}) = \underline{v}^{(k)}(\delta_{2,k}), \end{array}$$

because

$$\int_0^1 s(r)[\overline{u}^{(k)}(r) - \overline{v}^{(k)}(r)]dr = [\overline{u}^{(k)}(\delta_{1,k}) - \overline{v}^{(k)}(\delta_{1,k})] \int_0^1 s(r)dr = 0,$$

$$\int_0^1 s(r)[\underline{u}^{(k)}(r) - \underline{v}^{(k)}(r)]dr = [\underline{u}^{(k)}(\delta_{2,k}) - \underline{v}^{(k)}(\delta_{2,k})] \int_0^1 s(r)dr = 0.$$

□

4 Properties of m-Source Distance

Some properties of the approximation operators are presented by Grzegorzewski and Mrówka [10]. In this section we consider some properties of the approximation operator suggested in Section 3.

Let $T_m : F(\mathbb{R}) \longrightarrow PF_m(\mathbb{R})$ be the approximation operator which produces the nearest approximation fuzzy number out of $PF_m(\mathbb{R})$ to a given original fuzzy number using Theorem 2.1.

Theorem 4.1. *The nearest approximation operator is an 1-cut invariance.*

Proof. It is a necessary condition for this approximation that

$$[T_m(\tilde{u})]^1 = [\tilde{u}]^1.$$

□

Theorem 4.2. *The nearest approximation operator is invariant to translations.*

Proof. Let a be a real number. Let $\tilde{u}$ denotes a fuzzy number. Then $\underline{u+a} = \underline{u} + a$ and $\overline{u+a} = \overline{u} + a$. Therefore

$$Val(T_m(\tilde{u}+a)) = Val(\tilde{u}+a) = Val(\tilde{u}) + a = Val(T_m(\tilde{u})) + a,$$

$$Amb(T_m(\tilde{u}+a)) = Amb(\tilde{u}+a) = Amb(\tilde{u}) = Amb(T_m(\tilde{u})) = Amb(T_m(\tilde{u})+a),$$

Also for $k = 1, 2, \ldots, m-1$ we have

$$V((T_m(\tilde{u}) + a)^{(k)}) = V((\tilde{u}+a)^{(k)}) = V(\tilde{u}^{(k)}) = V(T_m(\tilde{u})^{(k)}),$$

and

$$A((T_m(\tilde{u}) + a)^{(k)}) = A((\tilde{u}+a)^{(k)}) = A(\tilde{u}^{(k)}) = A(T_m(\tilde{u})^{(k)}).$$

Thus

$$D_m(T_m(\tilde{u}+a), T_m(\tilde{u}) + a) = 0.$$

Since both $T_m(\tilde{u}+a)$ and $T_m(\tilde{u})+a$ have an m-degree polynomial form, then, from Lemma 3.6, we have

$$T_m(\tilde{u}+a) = T_m(\tilde{u}) + a.$$

□

Theorem 4.3. *The nearest approximation operator is scale invariant.*

Proof. Let $\lambda \neq 0$ be a real number. Thus

$$Val(\lambda\tilde{u}) = \lambda Val(\tilde{u}) \quad , \quad Amb(\lambda\tilde{u}) = \lambda Amb(\tilde{u}).$$

also for $k = 1, 2, \ldots, m-1$ we have

$$V(\lambda\tilde{u}^{(k)}) = \lambda V(\tilde{u}^{(k)}) \quad , \quad A(\lambda\tilde{u}^{(k)}) = \lambda A(\tilde{u}^{(k)}).$$

and

$$D_m(T_m(\lambda\tilde{u}), \lambda T_m(\tilde{u})) = 0.$$

Therefore then, from Lemma 3.6, we have

$$T_m(\lambda\tilde{u}) = \lambda T_m(\tilde{u}).$$

□

Theorem 4.4. *The nearest approximation operator fulfills the nearness criterion with respect to $m-$source metric D_m defined in Definition 3.1, on the set of all $m-$degree polynomial form fuzzy numbers.*

Proof. By Lemma 3.2, we have

$$D_m(\tilde{u}, T_m(\tilde{u})) = \min_{\tilde{v} \in PF_m(\mathbb{R})} D_m(\tilde{u}, \tilde{v}),$$

therefore

$$D_m(\tilde{u}, T_m(\tilde{u})) \leq D_m(\tilde{u}, \tilde{v}), \qquad \forall \tilde{v} \in PF_m(\mathbb{R}).$$

□

Theorem 4.5. *The nearest approximation operator is continuous.*

Proof. An approximation operator T is continuous if for any $\tilde{u}, \tilde{v} \in F(\mathbb{R})$ we have

$$\forall \epsilon > 0, \exists \delta > 0, D_m(\tilde{u}, \tilde{v}) < \delta \Longrightarrow D_m(T(\tilde{u}), T(\tilde{v})) < \epsilon.$$

Let $D_m(\tilde{u}, \tilde{v}) < \delta$. By Theorem 3.1 we have

$$D_m(T_m(\tilde{u}), T_m(\tilde{v})) \leq D_m(T_m(\tilde{u}), \tilde{u}) + D_m(\tilde{u}, \tilde{v}) + D_m(\tilde{v}, T_m(\tilde{v}))$$

and, by Theorem 3.3, we have $D_m(T_m(\tilde{u}), \tilde{u}) = D_m(\tilde{v}, T_m(\tilde{v})) = 0$. Thus

$$D_m(T_m(\tilde{u}), T_m(\tilde{v})) \leq D_m(\tilde{u}, \tilde{v}) < \delta.$$

Therefore it suffices to take $\delta \leq \epsilon$. □

Theorem 4.6. *The nearest trapezoidal approximation operator (case m=1) is monotonic on any set of fuzzy numbers with equal cores.*

Proof. See [1]. □

Theorem 4.7. *The nearest approximation operator is order invariant with respect to value function.*

Proof. The proof is trivial, because $Val(T_m(\tilde{u})) = Val(\tilde{u})$ and $Val(T_m(\tilde{v})) = Val(\tilde{v})$. □

Theorem 4.8. *The nearest approximation operator does not change the distance of fuzzy numbers by m-source distance. i.e.*

$$D_m(T_m(\tilde{u}), T_m(\tilde{v})) = T_m(D_m(\tilde{u}, \tilde{v})) = D_m(\tilde{u}, \tilde{v}).$$

Proof. The proof is trivial by Lemma 3.7. □

By the following Theorem we show that the nearest approximation operator is linear:

Theorem 4.9. *The nearest approximation operator is a linear operator on the set of all fuzzy numbers. i.e. for a real number* λ *and two fuzzy numbers* $\tilde{u}$ *and* $\tilde{v}$ *we have*

$$T_m(\lambda\tilde{u}+\tilde{v}) = \lambda T_m(\tilde{u}) + T_m(\tilde{v}).$$

Proof. From Lemma 3.1, we have

$$\begin{aligned} D_m(\lambda\tilde{u}+\tilde{v}, \lambda T_m(\tilde{u}) + T_m(\tilde{v})) &\leq D_m(\lambda\tilde{u}, \lambda T_m(\tilde{u})) + D_m(\tilde{v}, T_m(\tilde{v})) \\ &= D_m(\lambda\tilde{u}, \lambda T_m(\tilde{u})) \\ &= |\lambda| D_m(\tilde{u}, T_m(\tilde{u})) = 0. \end{aligned}$$

Also we have

$$D_m(\lambda\tilde{u}+\tilde{v}, T_m(\lambda\tilde{u}+\tilde{v})) = 0,$$

therefore

$$D_m(T_m(\lambda\tilde{u}+\tilde{v}), \lambda T_m(\tilde{u}) + T_m(\tilde{v})) = 0.$$

Since both fuzzy numbers $T_m(\lambda\tilde{u}+\tilde{v})$ and $\lambda T_m(\tilde{u})+T_m(\tilde{v})$ belong to $PF_m(\mathbb{R})$, we have

$$T_m(\lambda\tilde{u}+\tilde{v}) = \lambda T_m(\tilde{u}) + T_m(\tilde{v}).$$

□

5 Conclusion

In this work we represent a polynomial parametric approximation of a fuzzy number which has been introduced in [1], and we presents some important properties of it.

References

[1] Abbasbandy S., Amirfakhrian M., The nearest approximation of a fuzzy quantity in parametric form, *Appl. Math. and Comput.*, **172** (2006) 624–632.

[2] S. Abbasbandy, M. Amirfakhrian, The nearest trapezoidal form of a generalized left right fuzzy number, *Int. J. Approx. Reason.* 43 (2006) 166-178

[3] S. Abbasbandy, B. Asady, The nearest trapezoidal fuzzy number to a fuzzy quantity, Appl. Math. Comput. 156 (2004) 381-386.

[4] A. I. Ban, Triangular and parametric approximations of fuzzy numbers-inadvertences and corrections, Fuzzy Sets and Systems, In Press, Corrected Proof, Available online 23 April 2009.

[5] A. Ban, Approximation of fuzzy numbers by trapezoidal fuzzy numbers preserving the expected interval, Fuzzy Sets and Systems, **159** (2008) 1327-1344.

[6] D. Dubois, H. Prade, Operations on fuzzy numbers, *Intenat. J. Systems Sci.* **9** (1978) 613–626.

[7] D. Dubois, H. Prade, Fuzzy Sets and Systems: Theory and Application, Academic Press, New York, 1980.

[8] M. Delgado, M. A. Vila, W. Voxman, On a canonical representation of fuzzy numbers, *Fuzzy Sets and Systems* **93** (1998) 125-135.

[9] P. Grzegorzewski, Nearest interval approximation of a fuzzy number, *Fuzzy Sets and Systems* **130** (2002) 321-330.

[10] P. Grzegorzewski, P. Mrówka, Trapezoidal approximations of fuzzy numbers, *Fuzzy Sets and Systems* 153 (2005), 115-135.

[11] P. Grzegorzewski, E. Mrówka, Trapezoidal approximations of fuzzy numbers , In: Fuzzy Sets and Systems IFSA 2003, T. Bilgic, B. De Baets, O. Kay nak (Eds.), Lecture Notes in Artificial Intelligence 2715, Springer, 2003, pp. 237-244.

[12] P. Grzegorzewski, Trapezoidal approximations of fuzzy numbers preserving the expected interval - Algorithms and properties, Fuzzy Sets and Systems, **159** (2008) 1354-1364.

[13] P. Grzegorzewski, E. Mrówka, Trapezoidal approximations of fuzzy numbers-revisited, Fuzzy Sets and Systems, **158** (2007) 757-768.

[14] Xinwang Liua, Hsinyi Lin, Parameterized approximation of fuzzy number with minimum variance weighting functions, *Mathematica and Computer Modelling*, **46** (2007), 1398-1409.

[15] M. Ma and A. Kandel and M. Friedman, A new approach for defuzzification, *Fuzzy Sets and Systems* **111** (2000) 351-356.

[16] M. Ma and A. Kandel and M. Friedman, Correction to "A new approach for defuzzification", *Fuzzy Sets and Systems* **128** (2002) 133-134.

[17] Efendi N. Nasibov, Sinem Peker, On the nearest parametric approximation of a fuzzy number, Fuzzy Sets and Systems, **159** (2008) 1365-1375.

[18] W. Voxman, Some remarks on distance between fuzzy numbers, *Fuzzy Sets and Systems* **100** (1998) 353-365.

[19] J.S. Yao,K. Wu, Ranking fuzzy numbers based on decomposition principle and signed distance, *Fuzzy Sets and Systems* **116** (2000) 275-288.

[20] Wenyi Zeng, Hongxing Li, Weighted triangular approximation of fuzzy numbers, *Int. J. Approx. Reason.*, **46** (2007) 137-150.

[21] Chi-Tsuen Yeh, Trapezoidal and triangular approximations preserving the expected interval, Fuzzy Sets and Systems, **159** (2008) 1345-1353.

[22] Chi-Tsuen Yeh, A note on trapezoidal approximations of fuzzy numbers Fuzzy Sets and Systems, **158** (2007) 747-754.

[23] Zimmermann H.J., Fuzzy Set Theory and Its Applications, 2^{nd} Edition, Kluwer Academic, Boston, 1991.

Azad University, Central Tehran Branch,
Department of Mathematics,
Payambar Complex, Shahrak Gharb, Tehran, Iran.
e-mail: majid@amirfakhrian.com, majiamir@yahoo.com.