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An examination of the fundamental solution and Green's function in orthotropic photothermoelastic with diffusion under the MGT model

Saurav Sharma

Abstract

The objective of this paper is to investigate the fundamental solution and Green's function in a semi-infinite orthotropic photothermoelastic diffusion medium that is based on the Moore-Gibson-Thompson heat equation (MGTPWD). First, we transform the governing equations into a two-dimensional format and then make dimensionless to derive the general solution for the MGTPWD model. Based on the general solution, nine new harmonic functions were used to build the fundamental solution and Green's function for a steady point heat source on the surface and inside of a semi-infinite material in the proposed model. The elementary functions are used to express the components of displacements, stress, temperature distribution, carrier density distribution and chemical potential. The physical field quantities (stress, temperature distribution, carrier density distribution and chemical potential) are computed numerically and presented graphically to depict diffusion impact. A unique case have been deduced and compared with earlier known results. The results acquired can be used to delineate a variety of semiconductor elements during the coupled photo thermoelastic impact and can also be applied in the material and engineering sciences.

Key Words: Photothermoelastic diffusion, Moore-Gibson-Thompson, Orthotropic, Fundamental solution, Green's function, Harmonic functions.
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1 Opening Statement

The past few years have seen the extensive development of new methods in the investigation of semiconductors and microelectronic structures by the fields of photoacoustic (PA) and photothermal (PT) science and technology. PA and PT techniques were recently established as diagnostic methods with high sensitivity to the dynamics of photoexcited carriers (Mandelis [16], Nikolic and Todorovic [22], Almond and Patel [3]. Photogeneration of electron-hole pairs, also known as the carriers-diffusion wave or plasma wave, may be a significant factor in PA and PT experiments for the majority of semiconductor materials. This wave is generated by an absorbed intensity modulated laser beam.

Numerous researchers have examined the distinct effects of the thermoelastic and electronic deformations in semiconductor media, disregarding the coupling between the plasma and the thermoelastic equations (McDonald and Wetsel [21], Stearns and Kino [34]). The theoretical analysis was presented by Todorovi [35, 36, 37] to characterize two phenomena that provide information about the properties of transport and carrier recombinations in the semiconducting medium.

Marin et al. [18] examined a problem in microstretch thermoelastic with two temperature. In the context of micropolar thermoelastic diffusion, Marin et al. [17] investigated the fundamental equations and conditions of the mixed initial boundary value problem. In the context of micropolar thermoelastic diffusion. Sharma et al. [31] investigated the fundamental solution and plane waves in electro-microstretch elastic solids. Sharma and Marin [27] examined the reflection and transmission of waves in micropolar thermoelastic media. Marin et al. [19] employed the fractional calculus with thermal relaxation periods to analyze a novel representation of the porothermoelastic model. Different types of vibrations are addressed in Vlase et al. [40], Vlase et al. [39], Scutaru et al. [15], Vlase et al. [38], Ailawalia et al. [2] and Katouzian et al. [8]. Sharma et al. [28] studied fundamental theorems and plane waves by using a novel mathematical formulation of temperature dependent thermoelastic diffusion with multi-phase delays. Kumar et al. [9] investigated the impact of dual phase latency caused by the reflection of plane waves that propagate in a swelling porous thermoelastic medium with an impedance boundary. Quintanilla [25, 26] established models based on MGT heat equation. Abouelregal et al. [1] investigated thermomechanical waves in photothermoelastic with rotation medium. The concept of thermoelasticity was employed by Marin et al. [20] to investigate a dipole fluid under a modified MGT by incorporating suitable initial conditions.

Diffusion is the spontaneous movement of particles from a high concentration region to a low concentration region. It is a response to a concentration gradient, which is the change in concentration as a result of a change in position. Thermal diffusion is employed to achieve isotope separation by transferring heat across a thin liquid or vapour. Nowacki [23, 24] developed coupled thermoelastic diffusion models. By modifying Fourier's and Fick's laws, Sherief et al. [32] developed a generalized theory of thermoelasticity that incorporates mass diffusion. Kumar and Kansal [12] developed the fundamental equations for generalized thermoelastic diffusion (G-L model) and examined Lamb waves. Kumar and Chawla [10, 11] investigated the fundamental solution and Greens function in orthotropic thermoelastic diffusion media. Hou et al. [6, 7] examined the Green's function for a steady point heat source in the interior of a semi-infinite thermoelastic material in two- and three-dimensional problems. The two-dimensional general solutions and fundamental solutions in orthotropic thermoelastic materials were presented by Hou et al. [5]. Chawla and Kamboj [4] investigated the fundamental solution in anisotropic thermoelastic media with mass diffusion and voids. Sharma and Khator [29, 30] investigated the challenges associated with power generation that are a result of renewable energy sources and also explored the planning of microgrids in the renewable inclusive prosumer market.

Nevertheless, the critical fundamental solution to the two-dimensional problem of a constant point heat source in orthotropic photothermoelastic diffusion material has not been addressed to date.This paper examines the fundamental solution and Green's function for a two-dimensional orthotropic photothermoelastic diffusion medium. The fundamental solutions and Green's function for a stable point heat source acting on the surface and in the interior of a semi-infinite material of the MGTPWD model are obtained by seven newly introduced harmonic functions.

2 Fundamental equations

In the absence of body forces, heat sources, and carrier photogeneration sources, the constitutive relations and field equations for orthotropic photothermoelastic diffusion materials are as follows:

$$t_{ij,j} = \rho \ddot{u}_i, \tag{1}$$
$$t_{ij} = C^o_{ijkl} e_{kl} - \gamma^t_{ij} T - \gamma^p_{ij} P - \gamma^n_{ij} N, \tag{2}$$
$$K_{ij} \dot{T_{ji}} - K^*_{ij} T_{,ji} + \frac{E_g}{\tau} \dot{T} = \left(1 + \tau_o \frac{\partial}{\partial t}\right) [\gamma^t_{ij} T_o \ddot{e} + l_1 T_o \ddot{T} + d T_o \ddot{P}], \tag{3}$$
$$D_{ij} \dot{P_{,ji}} + D^p_{ij} P_{ji} = \left(1 + \tau_1 \frac{\partial}{\partial t}\right) [n \ddot{P} + l \gamma^p_{ij} \ddot{e} + d \ddot{T}], \tag{4}$$
$$D^*_{ij} N_{,ij} = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \zeta \frac{T}{\tau}, \qquad (i,j,k,l = 1,2,3). \tag{5}$$
$$where \tag{6}$$
$$C^o_{ijkl} = C_{ijkl} - \frac{b_{ij} b_{kl}}{b}, \quad \gamma^t_{ij} = \beta_{ij} + d b_{ij}, \quad \gamma^p_{ij} = \frac{b_{ij}}{b}, \quad \gamma^n_{ij} = C_{ijkl} \gamma^{n*}_{kl},$$
$$K_{ij} \delta_{ij} = K_i, \quad K^*_{ij} = K^*_i, \quad l_1 = \frac{\rho C_e}{T_o} + ad, \quad d = \frac{a}{b}, \quad n = \frac{1}{b},$$
$$D^*_{ij} \delta_{ij} = D^*_i. \ i - \ is\ not\ summed.$$

Also, t_{ij} are the components of stress tensor, u_i are the components of displacement, ρ is the medium density, C_{ijkl} are the elastic parameters, e_{kl} are the components of strain tensor, γ^t_{ij} are the coefficients of linear thermal expansion, γ^p_{ij} are the coefficients of diffusion, γ^n_{ij} are the coefficients of electronic deformation, $K_{(ij)}$ thermal conductivity, K^*_{ij} thermal conductivity rate, E_g denotes the semiconductor energy gap, T is the temperature distribution, N = n-n_o,(where n_o) is the carrier concentration at equilibrium, τ_o is the thermal relaxation time, T_o is the reference temperature, τ_1 represents the diffusion relaxation time, b_{ij} represents the components of diffusion tensor, C_e represents the specific heat, D^*_{ij} represents the coefficients of carrier diffusion, a and b are diffusion parameters, τ denotes the photogenerated carrier lifetime, $\zeta = \dfrac{\partial n_o}{\partial T}$, thermal activation coupling parameter. Partial derivatives and time derivatives are represented by the symbols "," and ".", respectively.

3 Problem Formulation

Based on the MGT model, we examine an orthotropic photothermoelastic solid with diffusion. Let OX_1 X_2 X_3 be the frame of reference in the Cartesian coordinate system, with the origin O being any point on the plane boundary. The plane of incidence for the two-dimensional static problem is assumed to be the consider X_1-X_3 plane, and our analysis is limited to this plane. Therefore, the temperature change, carrier density distribution, and displacement components are all functions of consider x_1 and consider x_3, with the following form:

$$\vec{u} = (u_1(x_1, 0, x_3), 0, u_3(x_1, 0, x_3)),\ T = T(x_1, x_3),$$
$$P = P(x_1, x_3), \quad and \quad N = N(x_1, x_3). \tag{7}$$

With the assistance of equation (7), we have derived the equations for orthotropic photothermoelastic solid in a two-dimensional problem by applying the appropriate plane of symmetry, as outlined in Slaughter [33], to the set of equations (1)-(5).

$$C_{11}^o \frac{\partial^2 u_1}{\partial x_1^2} + C_{55}^o \frac{\partial^2 u_1}{\partial x_3^2} + (C_{13}^o + C_{55}^o) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \gamma_1^t \frac{\partial T}{\partial x_1} - \gamma_1^p \frac{\partial P}{\partial x_1} - \gamma_1^n \frac{\partial N}{\partial x_1} = 0, \tag{8}$$

$$(C_{13}^o + C_{55}^o) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + C_{55}^o \frac{\partial^2 u_3}{\partial x_1^2} + C_{33}^o \frac{\partial^2 u_3}{\partial x_3^2} - \gamma_3^t \frac{\partial T}{\partial x_3} - \gamma_3^p \frac{\partial P}{\partial x_3} - \gamma_3^n \frac{\partial N}{\partial x_3} = 0, \tag{9}$$

$$K_1^* \frac{\partial^2 T}{\partial x_1^2} + K_3^* \frac{\partial^2 T}{\partial x_3^2} = 0, \quad D_1^p \frac{\partial^2 P}{\partial x_1^2} + D_3^p \frac{\partial^2 P}{\partial x_3^2} = 0,$$
$$D_1^* \frac{\partial^2 N}{\partial x_1^2} + D_3^* \frac{\partial^2 N}{\partial x_3^2} - \frac{N}{\tau} + \zeta \frac{T}{\tau} = 0, \tag{10}$$

with

$$t_{11} = C_{11}^o \frac{\partial u_1}{\partial x_1} + C_{13}^o \frac{\partial u_3}{\partial x_3} - \gamma_1^t T - \gamma_1^p P - \gamma_1^n N,$$
$$t_{33} = C_{13}^o \frac{\partial u_1}{\partial x_1} + C_{33}^o \frac{\partial u_3}{\partial x_3} - \gamma_3^t T - \gamma_3^p P - \gamma_3^n N, \tag{11}$$

$$t_{31} = C_{55}^o \left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right). \tag{12}$$

And

$$C_{11}^o = C_{11} - \frac{b_1^2}{b}, \quad C_{33}^o = C_{33} - \frac{b_3^2}{b}, \quad C_{13}^o = C_{13}, \quad C_{55}^o = C_{55},$$
$$\beta_1 = C_{11}\alpha_1^t + C_{12}\alpha_2^t + C_{13}\alpha_3^t, \tag{13}$$

$$\beta_3 = C_{13}\alpha_1^t + C_{23}\alpha_2^t + C_{33}\alpha_3^t,\ b_1 = C_{11}\alpha_1^c + C_{12}\alpha_2^c + C_{13}\alpha_3^c,$$
$$b_3 = C_{13}\alpha_1^c + C_{23}\alpha_2^c + C_{33}\alpha_3^c, \tag{14}$$

$$\gamma_1^n = C_{11}\gamma_1^{n^*} + C_{12}\gamma_2^{n^*} + C_{13}\gamma_3^{n^*}, \quad \gamma_3^n = C_{13}\gamma_1^{n^*} + C_{23}\gamma_2^{n^*} + C_{33}\gamma_3^{n^*}, \tag{15}$$

$\alpha_1^t,\ \alpha_2^t,\ \alpha_3^t;\ \alpha_1^c,\ \alpha_2^c,\ \alpha_3^c$ and $\gamma_1^{n^*},\ \gamma_2^{n^*},\ \gamma_3^{n^*}$ are linear thermal expansion coefficients, diffusion coefficients and electronic deformation coefficients, respectively. We define the dimensionless quantities

$$(x_1^{'}, x_3^{'}, u_1^{'}, u_3^{'}) = \frac{1}{L}(x_1, x_3, u_1, u_3), \quad (t_{11}^{'}, t_{33}^{'}, t_{31}^{'}) = \frac{1}{C_{11}^o}(t_{11}, t_{33}, t_{31}),$$
$$T^{'} = \frac{\gamma_1^t T}{C_{11}^o}, \quad N^{'} = \frac{N}{n_o}, \quad P^{'} = \frac{P}{b\gamma_1^p}. \tag{16}$$

Using the dimensionless quantities defined by equation (16) in equations (8)-(12) and, after suppressing the primes, this yields

$$\frac{\partial^2 u_1}{\partial x_1^2} + M_1\frac{\partial^2 u_1}{\partial x_3^2} + M_2\frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \frac{\partial T}{\partial x_1} - M_3\frac{\partial P}{\partial x_1} - M_4\frac{\partial N}{\partial x_1} = 0, \tag{17}$$
$$M_2\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + M_1\frac{\partial^2 u_3}{\partial x_1^2} + M_5\frac{\partial^2 u_3}{\partial x_3^2} - M_6\frac{\partial T}{\partial x_3} - M_7\frac{\partial P}{\partial x_3} - M_8\frac{\partial N}{\partial x_3} = 0, \tag{18}$$
$$\frac{\partial^2 T}{\partial x_1^2} + K^*\frac{\partial^2 T}{\partial x_3^2} = 0, \quad \frac{\partial^2 P}{\partial x_1^2} + D^*\frac{\partial^2 P}{\partial x_3^2} = 0,$$
$$\frac{\partial^2 N}{\partial x_1^2} + \bar{D}\frac{\partial^2 N}{\partial x_3^2} - M_9\frac{N}{\tau} + M_{10}\zeta\frac{T}{\tau} = 0, \tag{19}$$
$$t_{11} = \frac{\partial u_1}{\partial x_1} + M_{11}\frac{\partial u_3}{\partial x_3} - T - M_3 P - M_4 N, \ t_{31} = M_1\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right), \tag{20}$$
$$t_{33} = M_{11}\frac{\partial u_1}{\partial x_1} + M_5\frac{\partial u_3}{\partial x_3} - M_6 T - M_7 P - M_8 N. \tag{21}$$

with

$$M_1 = \frac{C_{55}^o}{C_{11}^o}, \quad M_2 = \frac{C_{13}^o + C_{55}^o}{C_{11}^o}, \quad M_3 = \frac{b\gamma_1^{2p}}{C_{11}^o}, \quad M_4 = \frac{n_o\gamma_1^n}{C_{11}^o},$$
$$M_5 = \frac{C_{55}^o}{C_{11}^o}, \frac{\gamma_3^t}{\gamma_1^t}, \quad M_7 = \frac{b\gamma_1^p\gamma_3^p}{C_{11}^o}, \quad M_8 = \frac{n_o\gamma_3^n}{C_{11}^o}, \quad M_9 = \frac{L^2}{\tau D_1^*},$$
$$M_{10} = \frac{\zeta L^2 C_{11}^o}{n_o\gamma_1^t\tau D_1^*}, \quad M_{11} = \frac{C_{13}^o}{C_{11}^o}, \quad K^* = \frac{K_3^*}{K_1^*}, \quad \bar{D} = \frac{D_3^*}{D_1^*}, \quad D^* = \frac{D_3^p}{D_1^p}. \tag{22}$$

Equations (17)-(20) can be written as:

$$D\left(u_1,\ u_3,\ T,\ P,\ N\right)^{tr} = 0 \tag{23}$$

Where tr denotes the transpose of D which is the differential operator matrix given by

$$\begin{bmatrix} \frac{\partial^2}{\partial x_1^2}+M_1\frac{\partial^2}{\partial x_3^2} & M_2\frac{\partial^2}{\partial x_1\partial x_3} & \frac{\partial}{\partial x_1} & -M_3\frac{\partial}{\partial x_1} & -M_4\frac{\partial}{\partial x_1} \\ M_2\frac{\partial^2}{\partial x_1\partial x_3} & M_1\frac{\partial^2}{\partial x_1^2}+M_5\frac{\partial^2}{\partial x_3^2} & -M_6\frac{\partial}{\partial x_3} & -M_7\frac{\partial}{\partial x_3} & -M_8\frac{\partial}{\partial x_3} \\ 0 & 0 & M_{10} & 0 & \frac{\partial^2}{\partial x_1^2}+\bar{D}\frac{\partial^2}{\partial x_3^2}-M_9 \\ 0 & 0 & \frac{\partial^2}{\partial x_1^2}+K^*\frac{\partial^2}{\partial x_3^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial^2}{\partial x_1^2}+D^*\frac{\partial^2}{\partial x_3^2} & 0 \end{bmatrix} \tag{24}$$

Equation (23) is a homogeneous set of differential equations in u_1, u_3, T, P, and N. The general solution by the operator theory is as follows

$$\begin{aligned} &u_1 = A_{i1}F + A^*_{i1}G, \quad u_3 = A_{i2}F + A^*_{i2}G, \quad T = A_{i3}F + A^*_{i3}G, \\ &P = A_{i4}F + A^*_{i4}G, \quad N = A_{i5}F + A^*_{i5}G, \quad (i = 1,2,3,4,5). \end{aligned} \tag{25}$$

Where A_{ij} (i,j=1,2,3,4) are algebraic cofactors of the matrix D, of which the determinant is

$$\begin{aligned} |D| = &\left(q_1\frac{\partial^4}{\partial x_3^4} - q_2\frac{\partial^4}{\partial x_1^2\partial x_3^2} + q_3\frac{\partial^4}{\partial x_1^4}\right)\times\left(\frac{\partial^2}{\partial x_1^2}+D^*\frac{\partial^2}{\partial x_3^2}\right) \\ &\times\left(\frac{\partial^2}{\partial x_1^2}+K^*\frac{\partial^2}{\partial x_3^2}\right)\times\left(\frac{\partial^2}{\partial x_1^2}+\bar{D}\frac{\partial^2}{\partial x_3^2}\right) \\ +&\left(q_1^\curlyvee\frac{\partial^4}{\partial x_3^4} - q_2^\curlyvee\frac{\partial^4}{\partial x_1^2\partial x_3^2} + q_3^\curlyvee\frac{\partial^4}{\partial x_1^4}\right)\times\left(\frac{\partial^2}{\partial x_1^2}+D^*\frac{\partial^2}{\partial x_3^2}\right)\times\left(\frac{\partial^2}{\partial x_1^2}+K^*\frac{\partial^2}{\partial x_3^2}\right). \end{aligned} \tag{26}$$

The functions F and G in equation (25) satisfy the following homogeneous equation

$$|D|F = 0 \quad and \quad |D|G = 0. \tag{27}$$

It is observed that the three sets of general solutions obtained with T=0 and P=0 are identical to those without thermal and diffusion effects if i is set to 1, 2, or 3 in equation (25). The general solution Y_1 (say) with P=0 is represented by i=4. The general solution Y_2 (for example) with T=0 is represented by i=5. The superposition of Y_1 and Y_2 is a result of the linear character of the photothermoelastic theory used in this paper.

$$
\begin{aligned}
u_1 &= \left(q_{11}\frac{\partial^6}{\partial x_1^6} + q_{12}\frac{\partial^6}{\partial x_1^4 \partial x_3^2} + q_{13}\frac{\partial^6}{\partial x_1^2 \partial x_3^4} + q_{14}\frac{\partial^6}{\partial x_3^6}\right)\frac{\partial F}{\partial x_1} \\
&\quad + \left(q_{11}^{\curlyvee}\frac{\partial^4}{\partial x_1^4} + q_{12}^{\curlyvee}\frac{\partial^4}{\partial x_1^2 \partial x_3^2} + q_{13}^{\curlyvee}\frac{\partial^4}{\partial x_3^4}\right)\frac{\partial G}{\partial x_1}, \qquad (28)\\
u_3 &= \left(q_{21}\frac{\partial^6}{\partial x_1^6} + q_{22}\frac{\partial^6}{\partial x_1^4 \partial x_3^2} + q_{23}\frac{\partial^6}{\partial x_1^2 \partial x_3^4} + q_{24}\frac{\partial^6}{\partial x_3^6}\right)\frac{\partial F}{\partial x_3} \\
&\quad + \left(q_{21}^{\curlyvee}\frac{\partial^4}{\partial x_1^4} + q_{22}^{\curlyvee}\frac{\partial^4}{\partial x_1^2 \partial x_3^2} + q_{23}^{\curlyvee}\frac{\partial^4}{\partial x_3^4}\right)\frac{\partial G}{\partial x_2}, \qquad (29)\\
T &= \left(q_{31}\frac{\partial^8}{\partial x_1^8} + q_{32}\frac{\partial^8}{\partial x_1^6 \partial x_3^2} + q_{33}\frac{\partial^8}{\partial x_1^4 \partial x_3^4} + q_{34}\frac{\partial^8}{\partial x_1^2 \partial x_3^6} + q_{35}\frac{\partial^8}{\partial x_3^8}\right)F \\
&\quad + \left(q_{31}^{\curlyvee}\frac{\partial^6}{\partial x_1^6} + q_{32}^{\curlyvee}\frac{\partial^6}{\partial x_1^4 \partial x_3^2} + q_{33}^{\curlyvee}\frac{\partial^6}{\partial x_1^2 \partial x_3^4} + q_{34}^{\curlyvee}\frac{\partial^6}{\partial x_3^6}\right)G, \qquad (30)\\
P &= \left(q_{41}\frac{\partial^8}{\partial x_1^8} + q_{42}\frac{\partial^8}{\partial x_1^6 \partial x_3^2} + q_{43}\frac{\partial^8}{\partial x_1^4 \partial x_3^4} + q_{44}\frac{\partial^8}{\partial x_1^2 \partial x_3^6} + q_{45}\frac{\partial^8}{\partial x_3^8}\right)F \\
&\quad + \left(q_{41}^{\curlyvee}\frac{\partial^6}{\partial x_1^6} + q_{42}^{\curlyvee}\frac{\partial^6}{\partial x_1^4 \partial x_3^2} + q_{43}^{\curlyvee}\frac{\partial^6}{\partial x_1^2 \partial x_3^4} + q_{44}^{\curlyvee}\frac{\partial^6}{\partial x_3^6}\right)G, \qquad (31)\\
N &= \left(q_{51}^{\curlyvee}\frac{\partial^6}{\partial x_1^6} + q_{52}^{\curlyvee}\frac{\partial^6}{\partial x_1^4 \partial x_3^2} + q_{53}^{\curlyvee}\frac{\partial^6}{\partial x_1^2 \partial x_3^4} + q_{54}^{\curlyvee}\frac{\partial^6}{\partial x_3^6}\right), \qquad (32)
\end{aligned}
$$

where

$q_{11}=M_1+M_1M_3,\quad q_{12}=M_5+M_3M_5-M_2M_6-M_2M_7+M_1D^*+M_1\bar{D}$
$+M_1M_3K^*+M_1M_3\bar{D},\quad q_{13}=(M_5-M_2M_6)\,D^*+M_3M_5K_*-M_2M_7K_*$
$+(M_5-M_2M_6+M_3M_5-M_2M_7)\,\bar{D}+M_1D_*\bar{D}+M_1M_3\bar{D}K^*,$
$q_{14}=(M_5-M_2M_6)\,D^*\bar{D}+(M_3M_5-M_2M_7)\,\bar{D}K^*,$
$q^*_{11}=-M_1M_9-M_1M_3M_9-M_1M_4M_{10},\quad q^*_{12}=M_2M_8M_{10}-M_5M_9$
$+M_2M_6M_9-M_3M_5M_9-M_2M_7M_9-M_4M_5M_{10}-M_1M_9D^*-M_1M_4M_{10}D^*$
$-M_1M_3M_9K^*,q^*_{13}=(M_2M_8-M_4M_5)M_{10}D^*-(M_5-M_2M_6)M_9D^*-$
$(M_3M_5-M_2M_7)M_9K^*,\ q_{21}=M_6+M_7-M_2-M_2M_3,q_{22}=M_1M_6+M_1M_7$
$+(M_6-M_2)\,D^*+(M_6+M_7-M_2-M_2M_3)\,\bar{D}+(M_7-M_2M_3)\,\bar{D}K^*,$
$q_{23}=M_1M_6D^*+(M_1M_6+M_1M_7)\,\bar{D}+M_1M_7K^*-M_2M_6D^*\bar{D}$
$+(M_7-M_2M_3)\,\bar{D}K^*,\ q_{24}=M_1M_6\bar{D}D^*+M_1M_7\bar{D}K^*,$
$q^*_{21}=M_6M_9-M_2M_9-M_7M_9-M_8M_{10}+M_2M_3M_9+M_2M_4M_{10},$
$q^*_{22}=-M_1M_6M_9-M_1M_8M_{10}+M_1M_7M_9-(M_7M_9-M_2M_3M_9)\,K^*$
$+(M_6M_9-M_2M_9-M_8M_{10}+M_2M_4M_{10})D^*,$
$q^*_{23}=(M_1M_6M_9-M_1M_8M_{10})\,D^*-M_1M_7M_9K^*,q_{31}=M_5=q_{41},$
$q_{32}=M_5+M_1^2-M_2^2+M_1D^*+M_1\bar{D},\ q_{33}=M_1M_5+(M_5+M_1^2-M_2^2)D^*$
$+\left(M_5+M_1^2-M_2^2\right)\bar{D}+M_1D^*\bar{D},$
$q_{34}=M_1M_5D^*+M_1M_5\bar{D}+\left(M_5+M_1^2-M_2^2\right)D^*\bar{D},$
$q_{35}=M_1M_5D^*\bar{D},q^*_{31}=-M_1M_9,\ q^*_{32}=-\left(M_5+M_1^2-M_2^2\right)M_9-M_1M_9D^*,$
$q^*_{33}=-M_1M_5M_9-(M_5+M_1^2-M_2^2)M_9D^*,q^*_{34}=-M_1M_5M_9D^*,$
$q_{42}=M_5+M_1^2-M_2^2+M_1\bar{D}+M_1K^*,$
$q_{43}=M_1M_5+(M_5+M_1^2-M_2^2)\bar{D}+(M_5+M_1^2-M_2^2)K^*+M_1\bar{D}K^*,$
$q_{44}=M_1M_5\bar{D}+M_1M_5K^*+(M_5+M_1^2-M_2^2)\bar{D}K^*,q_{45}=M_1M_5\bar{D}K^*,$
$q^*_{41}=-M_1M_9,q^*_{42}=-\left(M_5+M_1^2-M_2^2\right)M_9-M_1M_9K^*,$
$q^*_{43}=-M_1M_5M_9-\left(M_5+M_1^2-M_2^2\right)M_9K^*,\ q^*_{44}=-M_1M_5M_9K^*,$
$q_{51}=-M_1M_{10},\ q_{52}=-M_1M_{10}D^*-\left(M_5+M_1^2-M_2^2\right)M_{10},$
$q_{53}=-M_1M_5M_{10}-\left(M_5+M_1^2-M_2^2\right)M_{10}D^*,\ q_{54}=-M_1M_5M_{10}D^*.$

F and G are general solutions of equation (27) respectively, which can be written as

$$\prod_{j=1}^{5}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_{3j}^2}\right)F=0,\quad \prod_{j=1}^{4}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_{3j}^{\curlyvee\,2}}\right)G=0, \tag{33}$$

where $x_{3j} = S_j x_3,\ S_3 = \sqrt{\dfrac{K_1^*}{K_3^*}},\ S_4 = \sqrt{\dfrac{D_1^p}{D_3^p}},\ S_5 = \sqrt{\dfrac{D_1^*}{D_3^*}}$ and S_j (j=1,2) are two roots (with positive real part) of the following algebraic equation

$$q_1 S^4 - q_2 S^2 + q_3 = 0. \tag{34}$$

and $x_{3j}^{\curlyvee} = S_j^{\curlyvee} x_3,\ S_3^{\curlyvee} = \sqrt{\dfrac{K_1^*}{K_3^*}},\ S_4^{\curlyvee} = \sqrt{\dfrac{D_1^p}{D_3^p}}$, and $S_j^{\curlyvee}$ (j=1,2) are two roots (with positive real part) of the following algebraic equation

$$q_1^{\curlyvee} S^4 - q_2^{\curlyvee} S^2 + q_3^{\curlyvee} = 0. \tag{35}$$

The functions F and G can be expressed in terms of harmonic functions, as is well-known from the generalized Almansi theorem.

1. $F = F_1 + F_2 + F_3 + F_4 + F_5,\ G = G_1 + G_2 + G_3 + G_4$
$$for\ distinct\ S_j (j = 1, 2, 3, 4, 5),\ S_j^{\curlyvee} (j = 1, 2, 3, 4). \tag{36}$$

2. $F = F_1 + F_2 + F_3 + F_4 + x_3 F_5,\ G = G_1 + G_2 + G_3 + x_3 G_4;$
$$S_1 \neq S_2 \neq S_3 \neq S_4 = S_5,\ S_1^{\curlyvee} \neq S_2^{\curlyvee} \neq S_3^{\curlyvee} = S_4^{\curlyvee}. \tag{37}$$

3. $F = F_1 + F_2 + F_3 + x_3 F_4 + x_3^2 F_5,\ G = G_1 + G_2 + x_3 G_3 + x_3^2 G_4;$
$$S_1 \neq S_2 \neq S_3 = S_4 = S_5,\ S_1^{\curlyvee} \neq S_2^{\curlyvee} = S_3^{\curlyvee} = S_4^{\curlyvee}. \tag{38}$$

4. $$F = F_1 + F_2 + x_3 F_3 + x_3^2 F_4 + x_3^3 F_5\ for\ S_1 \neq S_2 = S_3 = S_4 = S_5. \tag{39}$$

5. $F = F_1 + x_3 F_2 + x_3^2 F_3 + x_3^3 F_4 + x_3^4 F_5, G = G_1 + x_3 G_2 + x_3^2 G_3 + x_3^3 G_4;$
$$S_1 = S_2 = S_3 = S_4 = S_5, S_1^{\curlyvee} = S_2^{\curlyvee} = S_3^{\curlyvee} = S_4^{\curlyvee}. \tag{40}$$

where F_j (j=1,2,3,4,5) and G_j (j=1,2,3,4) satisfy the following harmonic equations

$$\begin{aligned} &\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_{3j}^2}\right) F_j = 0, \quad (j = 1, 2, 3, 4, 5), \\ &\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_{3j}^{\curlyvee\,2}}\right) G_j = 0, \quad (j = 1, 2, 3, 4) \end{aligned} \tag{41}$$

For the case of distinct roots, the general solution can be derived as follows

$$u_1 = \sum_{j=1}^{5} \frac{\partial \Psi_j}{\partial x_1} + \sum_{j=1}^{4} \frac{\partial \Psi_j^{\curlyvee}}{\partial x_1}, \quad u_3 = \sum_{j=1}^{5} S_j M_{1j} \frac{\partial \Psi_j}{\partial x_1} + \sum_{j=1}^{4} S_j^{\curlyvee} M_{1j}^{\curlyvee} \frac{\partial \Psi_j^{\curlyvee}}{\partial x_1},$$

$$T = \sum_{j=1}^{5} M_{2j} \frac{\partial^2 \Psi_j}{\partial x_{3j}^2} + \sum_{j=1}^{4} M_{2j}^{\curlyvee} \frac{\partial^2 \Psi_j^{\curlyvee}}{\partial x_{3j}^{\curlyvee\,2}},$$

$$P=\sum_{j=1}^{5} M_{3j}\frac{\partial^2\Psi_j}{\partial x_{3j}^2}+\sum_{j=1}^{4} M_{3j}^{\curlyvee}\frac{\partial^2\Psi_j^{\curlyvee}}{\partial {x_{3j}^{\curlyvee}}^2},\quad N=\sum_{j=1}^{5} M_{4j}\frac{\partial^2\Psi_j}{\partial x_{3j}^2},\tag{42}$$

with

$$\Psi_j=m_{1j}\frac{\partial^6 F_j}{\partial x_{3j}^6},\qquad \Psi_j^{\curlyvee}=m_{1j}^{\curlyvee}\frac{\partial^4 G_j}{\partial {x_{3j}^{\curlyvee}}^4}\tag{43}$$

$$\begin{aligned}
&M_{1j}=\frac{m_{2j}}{m_{1j}},\quad M_{1j}^{\curlyvee}=\frac{m_{2j}^{\curlyvee}}{m_{1j}^{\curlyvee}},\quad M_{23}=\frac{m_{33}}{m_{11}},M_{23}^{\curlyvee}=\frac{m_{33}^{\curlyvee}}{m_{13}^{\curlyvee}},\quad M_{34}=\frac{m_{44}}{m_{14}},\\
&M_{34}^{\curlyvee}=\frac{m_{44}^{\curlyvee}}{m_{14}^{\curlyvee}},\quad M_{4j}=\frac{m_{5j}}{m_{1j}},M_{21}=M_{22}=M_{24}=M_{25}=M_{21}^{\curlyvee}=M_{22}^{\curlyvee}=0,\\
&M_{24}^{\curlyvee}=M_{31}=M_{32}=M_{33}=M_{34}=M_{31}^{\curlyvee}=M_{32}^{\curlyvee}=M_{33}^{\curlyvee}=0.\\
&m_{kj}=-q_{k1}+q_{k2}S_j^2-q_{k3}S_j^4+q_{k4}S_j^6,\ (k=1,2,5),\\
&m_{kj}^{\curlyvee}=q_{k1}^{\curlyvee}-q_{k2}^{\curlyvee}{S_j^{\curlyvee}}^2+q_{k3}^{\curlyvee}{S_j^{\curlyvee}}^4,\ (k=1,2),\\
&m_{33}=q_{31}-q_{32}S_j^2+q_{33}S_j^4-q_{34}S_j^6+q_{35}S_j^8,\\
&m_{33}^{\curlyvee}=-q_{31}^{\curlyvee}+q_{32}^{\curlyvee}{S_j^{\curlyvee}}^2-q_{33}^{\curlyvee}{S_j^{\curlyvee}}^4+q_{34}^{\curlyvee}{S_j^{\curlyvee}}^6,\\
&m_{44}=q_{41}-q_{42}S_j^2+q_{43}S_j^4-q_{44}S_j^6+q_{45}S_j^8,\\
&m_{44}^{\curlyvee}=-q_{41}^{\curlyvee}+q_{42}^{\curlyvee}{S_j^{\curlyvee}}^2-q_{43}^{\curlyvee}{S_j^{\curlyvee}}^4+q_{44}^{\curlyvee}{S_j^{\curlyvee}}^6.
\end{aligned}$$

The functions Ψ_j (j=1,2,3,4,5) and $\Psi_j^{\curlyvee}$ (j=1,2,3,4) satisfy the harmonic equations

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_{3j}^2}\right)\Psi_j=0,\quad \left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial {x_{3j}^{\curlyvee}}^2}\right)\Psi_j^{\curlyvee}=0.\tag{44}$$

With the aid of equations (20)-(21) and (42), stress components t_{11}, t_{31}, t_{33} can be written as

$$
\begin{aligned}
t_{11} = \sum_{j=1}^{5} \Bigg(& -1 + N_{11} S_j^2 M_{1j} - M_{2j} - M_3 M_{3j} \\
& - \frac{M_4}{M_9} \left\{ M_{4j} \left(-1 + \bar{D} S_j^2 \right) + M_{10} M_{2j} \right\} \Bigg) \frac{\partial^2 \Psi_j}{\partial x_{3j}^2} \\
& + \sum_{j=1}^{4} \left(-1 + N_{11} {S_j^{\Upsilon}}^2 M_{1j}^{\Upsilon} - M_{2j}^{\Upsilon} - M_3 M_{3j}^{\Upsilon} - \frac{M_4 M_{10}}{M_9} M_{2j}^{\Upsilon} \right) \frac{\partial^2 \Psi_j^{\Upsilon}}{\partial {x_{3j}^{\Upsilon}}^2},
\end{aligned}
\tag{45}
$$

$$
t_{31} = \sum_{j=1}^{5} M_1 \left(1 + M_{1j} \right) S_j \frac{\partial^2 \Psi_j}{\partial x_1 \partial x_{3j}} + \sum_{j=1}^{4} M_1 \left(1 + M_{1j}^{\Upsilon} \right) S_j^{\Upsilon} \frac{\partial^2 \Psi_j^{\Upsilon}}{\partial x_1 \partial x_{3j}^{\Upsilon}},
\tag{46}
$$

$$
\begin{aligned}
t_{33} = \sum_{j=1}^{5} \Bigg(& -N_{11} + M_5 S_j^2 M_{1j} - M_6 M_{2j} - M_7 M_{3j} \\
& - \frac{M_8}{M_9} \left\{ M_{4j} \left(-1 + \bar{D} S_j^2 \right) + M_{10} M_{2j} \right\} \Bigg) \frac{\partial^2 \Psi_j}{\partial x_{3j}^2} \\
& + \sum_{j=1}^{4} \left(-N_{11} + M_5 {S_j^{\Upsilon}}^2 M_{1j}^{\Upsilon} - M_6 M_{2j}^{\Upsilon} - M_7 M_{3j}^{\Upsilon} - \frac{M_8 M_{10}}{M_9} M_{2j} \right) \frac{\partial^2 \Psi_j^{\Upsilon}}{\partial {x_{3j}^{\Upsilon}}^2}.
\end{aligned}
\tag{47}
$$

Substituting the values of t_{11}, t_{33} and t_{31} from equation (45), (46) and (47) in equations (1) yield

$$\begin{aligned} M_1\left(1+M_{1j}\right)S_j^2 &= 1 - N_{11}S_j^2 M_{1j} + M_{2j} + M_3 M_{3j} \\ &\quad + \frac{M_4}{M_9}\left\{M_{4j}\left(-1+\bar{D}S_j^2\right) + M_{10}M_{2j}\right\},\ (j=1,2,3,4,5) \end{aligned} \tag{48}$$

$$\begin{aligned} M_1\left(1+M_{1j}^{\curlyvee}\right)S_j^{\curlyvee\,2} &= 1 - N_{11}S_j^{\curlyvee\,2}M_{1j}^{\curlyvee} + M_3 M_{3j}^{\curlyvee} + \\ &\quad \left(1+\frac{M_4 M_{10}}{M_9}\right)M_{2j}^{\curlyvee},\ (j=1,2,3,4) \end{aligned} \tag{49}$$

$$\begin{aligned} M_1\left(1+M_{1j}\right) &= -N_{11} + M_5 S_j^2 M_{1j} - M_6 M_{2j} - M_7 M_{3j} \\ &\quad - \frac{M_8}{M_9}\left\{M_{4j}\left(-1+\bar{D}S_j^2\right) + M_{10}M_{2j}\right\},(j=1,2,3,4,5) \end{aligned} \tag{50}$$

$$\begin{aligned} M_1\left(1+M_{1j}^{\curlyvee}\right) &= -N_{11} + M_5 S_j^{\curlyvee\,2}M_{1j}^{\curlyvee} - M_7 M_{3j}^{\curlyvee} \\ &\quad - \left(M_6 + \frac{M_8 M_{10}}{M_9}\right)M_{2j},\ (j=1,2,3,4) \end{aligned} \tag{51}$$

The expressions of t_{11}, t_{33}, t_{31} given by (45)-(47) with the help of (48)-(51) can be simplified as

$$\begin{aligned} t_{11} &= -\sum_{j=1}^{5} S_j^2 \check{M}_{1j}\frac{\partial^2\Psi_j}{\partial x_{3j}^2} - \sum_{j=1}^{4} S_j^{\curlyvee\,2}\check{M}_{1j}^{\curlyvee}\frac{\partial^2\Psi_j^{\curlyvee}}{\partial x_{3j}^{\curlyvee\,2}}, \\ t_{33} &= \sum_{j=1}^{5}\check{M}_{1j}\frac{\partial^2\Psi_j}{\partial x_{3j}^2} + \sum_{j=1}^{4}\check{M}_{1j}^{\curlyvee}\frac{\partial^2\Psi_j^{\curlyvee}}{\partial x_{3j}^{\curlyvee\,2}}, \\ t_{31} &= \sum_{j=1}^{5} S_j\check{M}_{1j}\frac{\partial^2\Psi_j}{\partial x_1\partial x_{3j}} + \sum_{j=1}^{4} S_j^{\curlyvee}\check{M}_{1j}^{\curlyvee}\frac{\partial^2\Psi_j^{\curlyvee}}{\partial x_1\partial x_{3j}^{\curlyvee}}, \end{aligned} \tag{52}$$

with

$$\check{M}_{1j} = -N_{11} + M_5 S_j^2 M_{1j} - M_6 M_{2j} - M_7 M_{3j} \\ - \frac{M_8}{M_9}\left\{M_{4j}\left(-1+\bar{D}S_j^2\right) + M_{10}M_{2j}\right\} = M_1\left(1+M_{1j}\right) \\ = \frac{1 - N_{11}S_j^2 M_{1j} + M_{2j} + M_3 M_{3j} + \frac{M_4}{M_9}\left\{M_{4j}\left(-1+\bar{D}S_j^2\right) + M_{10}M_{2j}\right\}}{S_j^2}, \tag{53}$$

$$\check{M}_{1j}^{\curlyvee} = -N_{11} + M_5 {S_j^{\curlyvee}}^2 M_{1j}^{\curlyvee} - M_7 M_{3j}^{\curlyvee} - \left(M_6 + \frac{M_8 M_{10}}{M_9}\right) M_{2j} \\ = M_1\left(1+M_{1j}^{\curlyvee}\right) = \frac{1 - N_{11}{S_j^{\curlyvee}}^2 M_{1j}^{\curlyvee} + M_3 M_{3j}^{\curlyvee} + \left(1 + \frac{M_4 M_{10}}{M_9}\right) M_{2j}^{\curlyvee}}{{S_j^{\curlyvee}}^2}. \tag{54}$$

4 Fundamental solution for a point heat source in a semi-infinite orthotropic photothermoelastic material with diffusion under the MGT model

Under the MGT model, we examine a semi-infinite orthotropic photothermoelastic diffusion material with $x_3 \geq 0$. The surface $x_3 = 0$ is thermally insulated and stress-free, and a point heat source H_1 and diffusion source H_2 are applied at the origin (Figure 1(a)). The general solutions provided by equations (42) and (52) are subsequently applied and derived in this section. Introduce the harmonic functions as

$$\Psi_j = A_j\left[\frac{1}{2}\left(x_{3j}^2 - x_1^2\right)\left(\log r_j - \frac{3}{2}\right) - x_1 x_{3j}\tan^{-1}\frac{x_1}{x_{3j}}\right],\ (j = 1,2,3,4,5) \tag{55}$$

$$\Psi_j^{\curlyvee} = A_j^{\curlyvee}\left[\frac{1}{2}\left({x_{3j}^{\curlyvee}}^2 - x_1^2\right)\left(\log r_j^{\curlyvee} - \frac{3}{2}\right) - x_1 x_{3j}^{\curlyvee}\tan^{-1}\frac{x_1}{x_{3j}^{\curlyvee}}\right],\ (j = 1,2,3,4) \tag{56}$$

where

$$r_j = \sqrt{x_1^2 + x_{3j}^2}, \qquad r_j^{\curlyvee} = \sqrt{x_1^2 + {x_{3j}^{\curlyvee}}^2}. \tag{57}$$

and A_j (j=1,2,3,4,5), $A_j^{\curlyvee}$ (j=1,2,3,4,5), are arbitrary constants to be determined.

Here $A_3^{\curlyvee}$ can be expressed as a linear combination of A_3 and $A_4^{\curlyvee}$ can be expressed as a linear combination of A_4

$$i.e., \qquad A_3^{\curlyvee} = \vartheta_1 A_3 \qquad and \qquad A_4^{\curlyvee} = \vartheta_2 A_4 \tag{58}$$

where ϑ_1 and ϑ_2 is an arbitrary constants.
The expressions for the displacement, temperature change, carrier density field, and stress components are obtained by substituting the values of Ψ_j and $\Psi_j^{\curlyvee}$ from equations (55) and (56) into equations (42) and (52).

$$\begin{aligned} u_1 = &-\sum_{j=1}^{5} A_j \left[x_1 (\log r_j - 1) + x_{3j} \tan^{-1} \frac{x_1}{x_{3j}}\right] \\ &-\sum_{j=1}^{4} A_j^{\curlyvee} \left[x_1 \left(\log r_j^{\curlyvee} - 1\right) + x_{3j}^{\curlyvee} \tan^{-1} \frac{x_1}{x_{3j}^{\curlyvee}}\right], \end{aligned} \tag{59}$$

$$\begin{aligned} u_3 = &\sum_{j=1}^{5} S_j M_{1J} A_j \left[x_{3j} (\log r_j - 1) - x_1 \tan^{-1} \frac{x_1}{x_{3j}}\right] \\ &+\sum_{j=1}^{4} S_j^{\curlyvee} M_{1J}^{\curlyvee} A_j^{\curlyvee} \left[x_{3j}^{\curlyvee} \left(\log r_j^{\curlyvee} - 1\right) - x_1 \tan^{-1} \frac{x_1}{x_{3j}^{\curlyvee}}\right], \end{aligned} \tag{60}$$

$$T = M_{23} A_3 \log r_3 + M_{23}^{\curlyvee} A_3^{\curlyvee} \log r_3^{\curlyvee}, \quad P = M_{34} A_4 \log r_4 + M_{34}^{\curlyvee} A_4^{\curlyvee} \log r_4^{\curlyvee}, \tag{61}$$

$$N = \sum_{j=1}^{5} M_{4j} A_j \left[\frac{1}{2}\left(x_{3j}^2 - x_1^2\right)\left(\log r_j - \frac{3}{2}\right) - x_1 x_{3j} \tan^{-1} \frac{x_1}{x_{3j}}\right],$$

$$t_{33} = \sum_{j=1}^{5} \check{M}_{1j} A_j \log r_j + \sum_{j=1}^{5} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} \log r_j^{\curlyvee}, \tag{62}$$

$$t_{11} = -\sum_{j=1}^{5} S_j^2 \check{M}_{1j} A_j \log r_j + \sum_{j=1}^{5} S_j^{\curlyvee\,2} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} \log r_j^{\curlyvee},$$

$$t_{31} = -\sum_{j=1}^{5} S_j \check{M}_{1j} A_j \tan^{-1} \frac{x_1}{x_{3j}} + \sum_{j=1}^{5} S_j^{\curlyvee} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} \tan^{-1} \frac{x_1}{x_{3j}^{\curlyvee}}. \tag{63}$$

5 Boundary Conditions

The boundary conditions at the stress-free surface $x_3 = 0$ are as follows:
The normal stress and tangential stress vanish, as well as the insulated thermal boundary and the gradient of diffusion and carried density field. In terms of mathematics, these are as follows:

$$t_{33}=0, \quad t_{31}=0, \quad \frac{\partial T}{\partial x_3}=0, \quad \frac{\partial P}{\partial x_3}=0, \quad \frac{\partial N}{\partial x_3}=0. \tag{64}$$

The following four conditions are obtained after the mechanical, thermal, diffusion, and conveying density field are taken into account for a rectangle with dimension of $0 \leq x_3 \leq a$ and $-b \leq x_1 \leq b$, $b > 0$ (Figure 1(a)).

$$\int_{-b}^{b} t_{33}\left(x_1, a\right) dx_1 + \int_0^a \left[t_{31}\left(b, x_3\right) - t_{31}\left(-b, x_3\right)\right] dx_3 = 0, \tag{65}$$

$$-K^* \int_{-b}^{b} \frac{\partial T}{\partial x_3}\left(x_1, a\right) dx_1 - \int_0^a \left[\frac{\partial T}{\partial x_1}\left(b, x_3\right) - \frac{\partial T}{\partial x_1}\left(-b, x_3\right)\right] dx_3 = H_1, \tag{66}$$

$$-D^* \int_{-b}^{b} \frac{\partial P}{\partial x_3}\left(x_1, a\right) dx_1 - \int_0^a \left[\frac{\partial P}{\partial x_1}\left(b, x_3\right) - \frac{\partial P}{\partial x_1}\left(-b, x_3\right)\right] dx_3 = H_2, \tag{67}$$

$$\int_{-b}^{b} \frac{\partial N}{\partial x_3}\left(x_1, a\right) dx_1 + \int_0^a \left[\frac{\partial N}{\partial x_1}\left(b, x_3\right) - \frac{\partial N}{\partial x_1}\left(-b, x_3\right)\right] dx_3 = 0. \tag{68}$$

Following the boundary conditions (64), we get the following when we substitute the values of t_{33}, t_{31}, T, P and N from equations (61), (62), and (63).

$$\begin{gathered}\sum_{j=1}^{5} \check{M}_{1j} A_j = 0, \quad \sum_{j=1}^{4} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} = 0, \quad \sum_{j=1}^{4} S_j \check{M}_{1j} A_j = 0, \\ \sum_{j=1}^{4} S_j^{\curlyvee} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} = 0, \quad \sum_{j=1}^{5} M_{4j} S_j A_j = 0. \end{gathered} \tag{69}$$

At the surface $x_3 = 0$, $\dfrac{\partial T}{\partial x_3}$ and $\dfrac{\partial P}{\partial x_3}$ are automatically satisfied. Substituting the values of t_{33} and t_{31} from equations (62) and (63) in equation (65), we obtain

$$\sum_{j=1}^{5} \check{M}_{1j} A_j I_1 + \sum_{j=1}^{4} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} I_2 = 0, \tag{70}$$

with

$$I_1 = \int_{-b}^{b} \log \sqrt{x_1^2 + S_j^2 a^2}\, dx_1 - 2\int_0^a S_j \tan^{-1}\left(\frac{b}{S_j x_3}\right) dx_3 = 2b\left(\log b - 1\right), \tag{71}$$

$$I_2 = \int_{-b}^{b} \log \sqrt{x_1^2 + S_j^{*2} a^2}\, dx_1 - 2\int_0^a S_j^{\curlyvee} \tan^{-1}\left(\frac{b}{S_j^{\curlyvee} x_3}\right) dx_3 = 2b\left(\log b - 1\right), \tag{72}$$

Now,

$$\int \frac{\partial T}{\partial x_3} dx_1 = M_{23} A_3 S_3 \tan^{-1}\left(\frac{x_1}{x_{33}}\right) + M_{23}^{\curlyvee} A_3^{\curlyvee} S_3^{\curlyvee} \tan^{-1}\left(\frac{x_1}{x_{33}^{\curlyvee}}\right), \quad (73)$$

$$\int \frac{\partial T}{\partial x_1} dx_3 = -\frac{M_{23} A_3}{S_3} \tan^{-1}\left(\frac{x_1}{x_{33}}\right) - \frac{M_{23}^{\curlyvee} A_3^{\curlyvee}}{S_3^{\curlyvee}} \tan^{-1}\left(\frac{x_1}{x_{33}^{\curlyvee}}\right), \quad (74)$$

$$\int \frac{\partial P}{\partial x_3} dx_1 = M_{34} A_4 S_4 \tan^{-1}\left(\frac{x_1}{x_{34}}\right) + M_{34}^{\curlyvee} A_4^{\curlyvee} S_4^{\curlyvee} \tan^{-1}\left(\frac{x_1}{x_{34}^{\curlyvee}}\right), \quad (75)$$

$$\int \frac{\partial P}{\partial x_1} dx_3 = -\frac{M_{34} A_4}{S_4} \tan^{-1}\left(\frac{x_1}{x_{34}}\right) - \frac{M_{34}^{\curlyvee} A_4^{\curlyvee}}{S_4^{\curlyvee}} \tan^{-1}\left(\frac{x_1}{x_{34}^{\curlyvee}}\right), \quad (76)$$

$$\int \frac{\partial N}{\partial x_3} dx_1 = \sum_{j=1}^{4} M_{4j} A_j S_j \left[x_1 x_{3j}\left(\log r_j - \frac{3}{2}\right) + \frac{1}{2}\left(x_{3j}^2 - x_1^2\right) \tan^{-1}\left(\frac{x_1}{x_{3j}}\right)\right], \quad (77)$$

$$\int \frac{\partial N}{\partial x_1} dx_3 = \sum_{j=1}^{4} \frac{M_{4j} A_j}{S_j} \left[x_1 x_{3j}\left(\log r_j - \frac{3}{2}\right) + \frac{1}{2}\left(x_{3j}^2 - x_1^2\right) \tan^{-1}\left(\frac{x_1}{x_{3j}}\right)\right]. \quad (78)$$

With the integrals (73) and (74) as a reference and $S_3 = \sqrt{\frac{K_1^*}{K_3^*}} = S_3^{\curlyvee}$, Equation (66) with substitution (61) yields

$$\check{M}_{23} A_3 I_3 + \check{M}_{23}^{\curlyvee} A_3^{\curlyvee} I_4 = \frac{H_1}{\sqrt{K_3^*/K_1^*}}, \quad (79)$$

$$\textit{with} \quad I_3 = -\left[\tan^{-1}\left(\frac{x_1}{S_3 a}\right)\right]_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{b}{S_3 x_3}\right)\right]_{x_3=0}^{x_3=a} = -\pi, \quad (80)$$

$$I_4 = -\left[\tan^{-1}\left(\frac{x_1}{S_3^{\curlyvee} a}\right)\right]_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{b}{S_3^{\curlyvee} x_3}\right)\right]_{x_3=0}^{x_3=a} = -\pi. \quad (81)$$

The constant A_3 is calculated by using equations (79), (80), (81) and (58) as

$$A_3 = -\frac{H_1}{\pi\left(\check{M}_{23} + \vartheta_1 \check{M}_{23}^{\curlyvee}\right)\sqrt{K_3^*/K_1^*}}. \quad (82)$$

Using $S_4 = \sqrt{\frac{D_1^p}{D_3^p}} = S_4^{\curlyvee}$, and the integrals (75) and (76) as a reference, substituting (61) in equation (67), yield

$$\check{M}_{34}A_4I_5+\check{M}_{34}^{\curlyvee}A_4^{\curlyvee}I_6=\frac{H_2}{\sqrt{D_3^p/D_1^p}},\tag{83}$$

where

$$I_5=-\left[\tan^{-1}\left(\frac{x_1}{S_4a}\right)\right]_{x_1=-b}^{x_1=b}+\left[\tan^{-1}\left(\frac{b}{S_4x_3}\right)\right]_{x_3=0}^{x_3=a}=-\pi,\tag{84}$$

$$I_6=-\left[\tan^{-1}\left(\frac{x_1}{S_4^{\curlyvee}a}\right)\right]_{x_1=-b}^{x_1=b}+\left[\tan^{-1}\left(\frac{b}{S_4^{\curlyvee}x_3}\right)\right]_{x_3=0}^{x_3=a}=-\pi.\tag{85}$$

The constant A_4 is calculated by using equations (83), (84), (85) and (58) as

$$A_4=-\frac{H_2}{\pi\left(\check{M}_{34}+\vartheta_2\check{M}_{34}^{\curlyvee}\right)\sqrt{D_3^p/D_1^p}}.\tag{86}$$

Using equation (61) to equation (68) and applying the integrals (77) and (78) provide

$$\sum_{j=1}^{5}M_{4j}A_j\wp_j=0.\tag{87}$$

where

$$\wp_j=\left(S_j^2-1\right)\left(ab\log\left(b^2+S_j^2a^2\right)-3ab+S_ja^2\tan^{-1}\left(\frac{b}{S_ja}\right)\right.$$
$$\left.-\frac{b^2}{S_ja}\tan^{-1}\left(\frac{b}{S_ja}\right)\right)-\frac{b^2\pi}{2S_j}.\tag{88}$$

Using the Cramer rule approach, seven equations (58), (69), (82), (86), and (87) can be used to find the remaining five constants, A_j (j=1,2,5), A_j^* (j=1,2):

$$\Delta=\begin{vmatrix}\check{M}_{11} & \check{M}_{12} & \check{M}_{15} & 0 & 0\\ S_1\check{M}_1 & S_2\check{M}_{12} & S_5\check{M}_{15} & 0 & 0\\ \wp_1\check{M}_{11} & \wp_2\check{M}_{12} & \wp_5\check{M}_{15} & 0 & 0\\ 0 & 0 & 0 & \check{M}_{11}^{\curlyvee} & \check{M}_{12}^{\curlyvee}\\ 0 & 0 & 0 & S_1^{\curlyvee}\check{M}_{11}^{\curlyvee} & S_2^{\curlyvee}\check{M}_{12}^{\curlyvee}\end{vmatrix}\neq 0,\tag{89}$$

$$and\quad A_1=\frac{\Delta_1}{\Delta}\quad A_2=\frac{\Delta_2}{\Delta}\quad A_3=\frac{\Delta_3}{\Delta}\quad A_1^{\curlyvee}=\frac{\Delta_4}{\Delta}\quad A_1^{\curlyvee}=\frac{\Delta_5}{\Delta}.\tag{90}$$

where Δ_1, Δ_2, Δ_3, Δ_4, Δ_5 are obtained by replacing the 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, 5^{th} column by

$$\begin{bmatrix}-\left(\check{M}_{13}A_3+\check{M}_{14}A_4\right)\\ -\left(S_3\check{M}_{13}A_3+S_4\check{M}_{14}A_4\right)\\ -\left(\wp_3M_{13}A_3+\wp_4M_{14}A_4\right)\\ -\left(\vartheta_1\check{M}_{13}A_3+\vartheta_2\check{M}_{14}A_4\right)\\ -\left(\vartheta_1S_3^{\curlyvee}\check{M}_{13}A_3+\vartheta_2S_4^{\curlyvee}\check{M}_{14}^{\curlyvee}A_4\right)\end{bmatrix}.\tag{91}$$

Where A_3 and A_4 are given by equations (82) and (86) and $A_3^{\curlyvee}$ and $A_4^{\curlyvee}$ is determined from equation (58).

6 Greens functions for a constant line heat source in a semi-infinite orthotropic photothermoelastic diffusion medium based on MGT model.

Under the MGT model, we examine a semi-infinite orthotropic photothermoelastic diffusion material with $x_3 \geq 0$. The surface $x_3 = 0$ is taken as stress-free, thermally insulated, and the gradient of diffusion and the carried density field has vanished. A point heat source of strength H_3 and a diffusion source H_4 are applied at the point $(0, h)$ in a two-dimensional Cartesian coordinate system (x_1, x_3) (Figure 1(b)). The following notations are introduced for future reference:

$$x_{3j} = S_j x_3, \quad h_k = S_k h, \quad x_{3j}^{\curlyvee} = S_j^{\curlyvee} x_3, \quad h_k^{\curlyvee} = S_k^{\curlyvee} h, \quad x_{3jk} = x_{3j} + h_k,$$

$$r_{jk} = \sqrt{x_1^2 + x_{3jk}^2}, \quad \bar{x}_{3jk} = x_{3j} - h_k, \quad \bar{r}_{jk} = \sqrt{x_1^2 + \bar{x}_{3jk}^2}, \quad x_{3jk}^{\curlyvee} = x_{3j}^{\curlyvee} + h_k^{\curlyvee},$$

$$r_{jk}^{\curlyvee} = \sqrt{x_1^2 + (x_{3jk}^{\curlyvee})^2}, \quad \bar{x}_{3jk}^{\curlyvee} = x_{3j}^{\curlyvee} - h_k^{\curlyvee}, \quad \bar{r}_{jk}^{\curlyvee} = \sqrt{x_1^2 + (\bar{x}_{3jk}^{\curlyvee})^2}. \tag{92}$$

Following Hou et al. [7], Greens functions in the semi-infinite plane are assumed to be of the following form:

$$\begin{aligned}\Psi_j = A_j &\left[\frac{1}{2}\left(\bar{x}_{3jj}^2 - x_1^2\right)\left(\log \bar{r}_{jj} - \frac{3}{2}\right) - x_1 \bar{x}_{3jj} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\ &+ \sum_{k=1}^{5} A_{jk}\left[\frac{1}{2}\left(x_{3jk}^2 - x_1^2\right)\left(\log r_{jk} - \frac{3}{2}\right) - x_1 x_{3jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right],\end{aligned} \tag{93}$$

$$\begin{aligned}\Psi_j^{\curlyvee} = A_j^{\curlyvee} &\left[\frac{1}{2}\left({\bar{x}_{3jj}^{\curlyvee}}^2 - x_1^2\right)\left(\log \bar{r}_{jj}^{\curlyvee} - \frac{3}{2}\right) - x_1 \bar{x}_{3jj}^{\curlyvee} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}^{\curlyvee}}\right)\right] \\ + \sum_{k=1}^{4} A_{jk}^{\curlyvee} &\left[\frac{1}{2}\left({x_{3jk}^{\curlyvee}}^2 - x_1^2\right)\left(\log r_{jk}^{\curlyvee} - \frac{3}{2}\right) - x_1 x_{3jk}^{\curlyvee} \tan^{-1}\left(\frac{x_1}{x_{3jk}^{\curlyvee}}\right)\right].\end{aligned} \tag{94}$$

Substituting the values of Ψ_j and Ψ_j^{Υ} in equations (42) and (52), we get:

$$\begin{aligned}
u_1 = &-\sum_{j=1}^{5} A_j \left[x_1 \left(\log \bar{r}_{jj} - 1\right) + \bar{x}_{3jj} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\
&-\sum_{j=1}^{5}\sum_{k=1}^{5} A_{jk} \left[x_1 \left(\log r_{jk} - 1\right) + x_{3jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right] \\
&-\sum_{j=1}^{4} A_j^{\Upsilon} \left[x_1 \left(\log \bar{r}_{jj}^{\Upsilon} - 1\right) + \bar{x}_{3jj}^{\Upsilon} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}^{\Upsilon}}\right)\right] \\
&-\sum_{j=1}^{4}\sum_{k=1}^{4} A_{jk}^{\Upsilon} \left[x_1 \left(\log r_{jk}^{\Upsilon} - 1\right) + x_{3jk}^{\Upsilon} \tan^{-1}\left(\frac{x_1}{x_{3jk}^{\Upsilon}}\right)\right],
\end{aligned} \tag{95}$$

$$\begin{aligned}
u_3 = &\sum_{j=1}^{5} S_j M_{1j} A_j \left[\bar{x}_{3jj} \left(\log \bar{r}_{jj} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\
&+\sum_{j=1}^{5}\sum_{k=1}^{5} S_j M_{1j} A_{jk} \left[x_{3jk} \left(\log r_{jk} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right] \\
&+\sum_{j=1}^{4} S_j^{\Upsilon} M_{1j}^{\Upsilon} A_j^{\Upsilon} \left[\bar{x}_{3jj}^{\Upsilon} \left(\log \bar{r}_{jj}^{\Upsilon} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}^{\Upsilon}}\right)\right] \\
&+\sum_{j=1}^{4}\sum_{k=1}^{4} S_j^{\Upsilon} M_{1j}^{\Upsilon} A_{jk}^{\Upsilon} \left[x_{3jk}^{\Upsilon} \left(\log r_{jk}^{\Upsilon} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{x_{3jk}^{\Upsilon}}\right)\right]
\end{aligned} \tag{96}$$

$$T = M_{23}A_3 \log \bar{r}_{33} + M_{23}\sum_{k=1}^{5} A_{3k} \log r_{3k} + M_{23}^{\curlyvee} A_3^{\curlyvee} \log \bar{r}_{33}^{\curlyvee} + M_{23}^{\curlyvee} \sum_{k=1}^{4} A_{3k}^{\curlyvee} \log r_{3k}^{\curlyvee}, \tag{97}$$

$$P = M_{34}A_4 \log \bar{r}_{44} + M_{34}\sum_{k=1}^{5} A_{4k} \log r_{4k} + M_{34}^{\curlyvee} A_4^{\curlyvee} \log \bar{r}_{44}^{\curlyvee} + M_{34}^{\curlyvee} \sum_{k=1}^{4} A_{4k}^{\curlyvee} \log r_{4k}^{\curlyvee}, \tag{98}$$

$$\begin{aligned} N = &\sum_{j=1}^{5} M_{4j}A_j \left[\frac{1}{2}\left(\bar{x}_{3jj}^2 - x_1^2\right)\left(\log \bar{r}_{jj} - \frac{3}{2}\right) - x_1 \bar{x}_{3jj} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\ &+ \sum_{j=1}^{5}\sum_{k=1}^{5} M_{4j}A_{jk} \left[\frac{1}{2}\left(x_{3jk}^2 - x_1^2\right)\left(\log r_{jk} - \frac{3}{2}\right) - x_1 x_{3jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right], \end{aligned} \tag{99}$$

$$\begin{aligned} t_{33} = &\sum_{j=1}^{5} \check{M}_{1j}A_j \log \bar{r}_{jj} + \sum_{j=1}^{5}\sum_{k=1}^{5} \check{M}_{1j}A_{jk} \log r_{jk} + \sum_{j=1}^{4} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} \log \bar{r}_{jj}^{\curlyvee} \\ &+ \sum_{j=1}^{4}\sum_{k=1}^{4} \check{M}_{1j}^{\curlyvee} A_{jk}^{\curlyvee} \log r_{jk}^{\curlyvee}, \end{aligned} \tag{100}$$

$$\begin{aligned} t_{11} = &-\sum_{j=1}^{5} S_j^2 \check{M}_{1j}A_j \log(\bar{r}_{jj}) - \sum_{j=1}^{5}\sum_{k=1}^{5} S_j^2 \check{M}_{1j}A_{jk} \log(r_{jk}) \\ &- \sum_{j=1}^{4} S_j^{\times\,2} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} \log(\bar{r}_{jj}^{\curlyvee}) - \sum_{j=1}^{4}\sum_{k=1}^{4} S_j^{\times\,2} \check{M}_{1j}^{\curlyvee} A_{jk}^{\curlyvee} \log(r_{jk}^{\curlyvee}), \end{aligned} \tag{101}$$

$$\begin{aligned} t_{31} = &-\sum_{j=1}^{5} S_j \check{M}_{1j}A_j \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right) - \sum_{j=1}^{5}\sum_{k=1}^{5} S_j \check{M}_{1j}A_{jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right) \\ &- \sum_{j=1}^{4} S_j^{\curlyvee} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}^{\curlyvee}}\right) - \sum_{j=1}^{4}\sum_{k=1}^{4} S_j^{\curlyvee} \check{M}_{1j}^{\curlyvee} A_{jk}^{\curlyvee} \tan^{-1}\left(\frac{x_1}{x_{3jk}^{\curlyvee}}\right). \end{aligned} \tag{102}$$

The following expressions emerge from the continuity at the plane $x_3 = h$ for u_3 and t_{31}.

$$\sum_{j=1}^{5} S_j M_{1j} A_j = 0, \quad \sum_{j=1}^{4} S_j^{\curlyvee} M_{1j}^{\curlyvee} A_j^{\curlyvee} = 0, \quad \sum_{j=1}^{5} S_j \check{M}_{1j} A_j = 0, \quad \sum_{j=1}^{4} S_j^{\curlyvee} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} = 0. \tag{103}$$

Substituting $\check{M}_{1j}$ and $\check{M}^{\curlyvee}_{1j}$ from Equations (53) and (54) into Equation (103), we get:

$$\sum_{j=1}^{5} S_j M_1(1+M_{1j})A_j = 0, \quad \sum_{j=1}^{4} S^{\curlyvee}_j M_1(1+M^{\curlyvee}_{1j})A^{\curlyvee}_j = 0. \tag{104}$$

Equation (104), by means of equation (102), can be written as

$$\sum_{j=1}^{5} S_j A_j = 0, \quad \sum_{j=1}^{4} S^{\curlyvee}_j A^{\curlyvee}_j = 0. \tag{105}$$

The following four conditions are obtained after the mechanical, thermal, diffusion, and carried density field are taken into account for a rectangle with the dimension $a_1 \le x_3 \le a_2$ $(0 < a_1 < h < a_2)$ and $-b \le x_1 \le b$.

$$\int_{-b}^{b} [t_{33}(x_1, a_2) - t_{33}(x_1, a_1)]\, dx_1 + \int_{a_1}^{a_2} [t_{31}(b, x_3) - t_{31}(-b, x_3)]\, dx_3 = 0, \tag{106}$$

$$\begin{aligned} &- K^* \int_{-b}^{b} \left[\frac{\partial T}{\partial x_3}(x_1, a_2) - \frac{\partial T}{\partial x_3}(x_1, a_1) \right] dx_1 \\ &\qquad - \int_{a_1}^{a_2} \left[\frac{\partial T}{\partial x_1}(b, x_3) - \frac{\partial T}{\partial x_1}(-b, x_3) \right] dx_3 = H_3, \end{aligned} \tag{107}$$

$$\begin{aligned} &- D^* \int_{-b}^{b} \left[\frac{\partial P}{\partial x_3}(x_1, a_2) - \frac{\partial P}{\partial x_3}(x_1, a_1) \right] dx_1 \\ &\qquad - \int_{a_1}^{a_2} \left[\frac{\partial P}{\partial x_1}(b, x_3) - \frac{\partial P}{\partial x_1}(-b, x_3) \right] dx_3 = H_4, \end{aligned} \tag{108}$$

$$\begin{aligned} &\int_{-b}^{b} \left[\frac{\partial N}{\partial x_3}(x_1, a_2) - \frac{\partial N}{\partial x_3}(x_1, a_1) \right] dx_1 \\ &\qquad + \int_{a_1}^{a_2} \left[\frac{\partial N}{\partial x_1}(b, x_3) - \frac{\partial N}{\partial x_1}(-b, x_3) \right] dx_3 = 0. \end{aligned} \tag{109}$$

Now

$$\int \log \bar{r}_{jj}\, dx_1 = x_1 \left(\log \bar{r}_{jj} - 1\right) + \bar{x}_{3jj} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right),$$

$$\int \log r_{jk}\, dx_1 = x_1 \left(\log r_{jk} - 1\right) + x_{3jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right), \tag{110}$$

$$\int \log \bar{r}^{\curlyvee}_{jj}\, dx_1 = x_1 \left(\log \bar{r}^{\curlyvee}_{jj} - 1\right) + \bar{x}^{\curlyvee}_{3jj} \tan^{-1}\left(\frac{x_1}{\bar{x}^{\curlyvee}_{3jj}}\right),$$

$$\int \log r^{\curlyvee}_{jk}\, dx_1 = x_1 \left(\log r^{\curlyvee}_{jk} - 1\right) + x^{\curlyvee}_{3jk} \tan^{-1}\left(\frac{x_1}{x^{\curlyvee}_{3jk}}\right), \tag{111}$$

$$\begin{aligned}\int \frac{\partial T}{\partial x_3}\, dx_1 = {}& S_3 M_{23} \left(A_3 \tan^{-1}\left(\frac{x_1}{\bar{x}_{333}}\right) + \sum_{k=1}^{5} A_{3k} \tan^{-1}\left(\frac{x_1}{x_{33k}}\right)\right) \\ & + S^{\curlyvee}_3 M^{\curlyvee}_{23} \left(A^{\curlyvee}_3 \tan^{-1}\left(\frac{x_1}{\bar{x}^{\curlyvee}_{333}}\right) + \sum_{k=1}^{4} A^{\curlyvee}_{3k} \tan^{-1}\left(\frac{x_1}{x^{\curlyvee}_{33k}}\right)\right),\end{aligned} \tag{112}$$

$$\begin{aligned}\int \frac{\partial T}{\partial x_1}\, dx_3 = {}& -\frac{M_{23}}{S_3} \left(A_3 \tan^{-1}\left(\frac{x_1}{\bar{x}_{333}}\right) + \sum_{k=1}^{5} A_{3k} \tan^{-1}\left(\frac{x_1}{x_{33k}}\right)\right) \\ & - \frac{M^{\curlyvee}_{23}}{S^{\curlyvee}_3} \left(A^{\curlyvee}_3 \tan^{-1}\left(\frac{x_1}{\bar{x}^{\curlyvee}_{333}}\right) + \sum_{k=1}^{4} A^{\curlyvee}_{3k} \tan^{-1}\left(\frac{x_1}{x^{\curlyvee}_{33k}}\right)\right),\end{aligned} \tag{113}$$

$$\begin{aligned}\int \frac{\partial P}{\partial x_3}\, dx_1 = {}& S_4 M_{34} \left(A_4 \tan^{-1}\left(\frac{x_1}{\bar{x}_{344}}\right) + \sum_{k=1}^{5} A_{4k} \tan^{-1}\left(\frac{x_1}{x_{34k}}\right)\right) \\ & + S^{\curlyvee}_4 M^{\curlyvee}_{34} \left(A^{\curlyvee}_4 \tan^{-1}\left(\frac{x_1}{\bar{x}^{\curlyvee}_{344}}\right) + \sum_{k=1}^{4} A^{\curlyvee}_{4k} \tan^{-1}\left(\frac{x_1}{x^{\curlyvee}_{34k}}\right)\right),\end{aligned} \tag{114}$$

$$\int \frac{\partial P}{\partial x_1}\,dx_3 = -\frac{M_{34}}{S_4}\left(A_4\tan^{-1}\left(\frac{x_1}{\bar{x}_{344}}\right)+\sum_{k=1}^{5}A_{4k}\tan^{-1}\left(\frac{x_1}{x_{34k}}\right)\right)$$
$$-\frac{M_{34}^{\curlyvee}}{S_4^{\curlyvee}}\left(A_4^{\curlyvee}\tan^{-1}\left(\frac{x_1}{\bar{x}_{344}^{\curlyvee}}\right)+\sum_{k=1}^{4}A_{4k}^{\curlyvee}\tan^{-1}\left(\frac{x_1}{x_{34k}^{\curlyvee}}\right)\right), \quad (115)$$

$$\int \frac{\partial N}{\partial x_3}\,dx_1 =$$
$$\sum_{j=1}^{5}M_{4j}S_jA_j\left[x_1\bar{x}_{3jj}\log\bar{r}_{jj}-\frac{3}{2}x_1\bar{x}_{3jj}+\frac{1}{2}\left(\bar{x}_{3jj}^2-x_1^2\right)\tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right]$$
$$+\sum_{j=1}^{5}\sum_{k=1}^{5}M_{4j}S_jA_{jk}\left[x_1x_{3jk}\log r_{jk}-\frac{3}{2}x_1x_{3jk}+\frac{1}{2}\left(x_{3jk}^2-x_1^2\right)\tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right], \quad (116)$$

$$\int \frac{\partial N}{\partial x_1}\,dx_3 =$$
$$\sum_{j=1}^{5}\frac{M_{4j}A_j}{S_j}\left[x_1\bar{x}_{3jj}\log\bar{r}_{jj}-\frac{3}{2}x_1\bar{x}_{3jj}+\frac{1}{2}\left(\bar{x}_{3jj}^2-x_1^2\right)\tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right]$$
$$+\sum_{j=1}^{5}\sum_{k=1}^{5}\frac{M_{3j}A_j}{S_j}\left[x_1x_{3jk}\log r_{jk}-\frac{3}{2}x_1x_{3jk}+\frac{1}{2}\left(x_{3jk}^2-x_1^2\right)\tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right]. \quad (117)$$

The integrals $\int_{a_1}^{a_2}\frac{\partial T}{\partial x_1}\,dx_3$, $\int_{a_1}^{a_2}\frac{\partial P}{\partial x_1}\,dx_3$, and $\int_{a_1}^{a_2}\frac{\partial N}{\partial x_1}\,dx_3$ in equations (113), (115), and (117) are not continuous at $x_3 = h$. Consequently, the following expressions should be used:

$$\int_{a_1}^{a_2}\frac{\partial T}{\partial x_1}\,dx_3 = \int_{a_1}^{h^-}\frac{\partial T}{\partial x_1}\,dx_3+\int_{h^+}^{a_2}\frac{\partial T}{\partial x_1}\,dx_3,$$
$$\int_{a_1}^{a_2}\frac{\partial P}{\partial x_1}\,dx_3 = \int_{a_1}^{h^-}\frac{\partial P}{\partial x_1}\,dx_3+\int_{h^+}^{a_2}\frac{\partial P}{\partial x_1}\,dx_3,$$
$$\int_{a_1}^{a_2}\frac{\partial N}{\partial x_1}\,dx_3 = \int_{a_1}^{h^-}\frac{\partial N}{\partial x_1}\,dx_3+\int_{h^+}^{a_2}\frac{\partial N}{\partial x_1}\,dx_3. \quad (118)$$

In equation (106), the values of t_{33} and t_{11} from equations (100) and (102) are substituted to determine:

$$\sum_{j=1}^{5} \check{M}_{1j} A_j I_7 + \sum_{j=1}^{5} \check{M}_{1j} \sum_{k=1}^{4} A_{jk} I_8 + \sum_{j=1}^{5} \check{M}_{1j}^{\curlyvee} A_j^{\curlyvee} I_9 + \sum_{j=1}^{5} \check{M}_{1j}^{\curlyvee} \sum_{k=1}^{4} A_{jk}^{\curlyvee} I_{10} = 0, \tag{119}$$

where

$$\begin{aligned} I_7 = &\left[x_1 \left(\log \bar{r}_{jj} - 1 \right) + \bar{x}_{3jj} \tan^{-1} \left(\frac{x_1}{\bar{x}_{3jj}} \right)_{x_3=a_1}^{x_3=a_2} \right]_{x_1=-b}^{x_1=b} \\ &- \left[x_1 \log \bar{r}_{jj} + \bar{x}_{3jj} \tan^{-1} \left(\frac{x_1}{\bar{x}_{3jj}} \right)_{x_1=-b}^{x_1=b} \right]_{x_3=a_1}^{x_3=a_2} = 0, \end{aligned} \tag{120}$$

$$\begin{aligned} I_8 = &\left[x_1 \left(\log r_{jk} - 1 \right) + x_{3jk} \tan^{-1} \left(\frac{x_1}{x_{3jk}} \right)_{x_3=a_1}^{x_3=a_2} \right]_{x_1=-b}^{x_1=b} \\ &- \left[x_1 \log r_{jk} + x_{3jk} \tan^{-1} \left(\frac{x_1}{x_{3jk}} \right)_{x_1=-b}^{x_1=b} \right]_{x_3=a_1}^{x_3=a_2} = 0, \end{aligned} \tag{121}$$

$$\begin{aligned} I_9 = &\left[x_1 \left(\log \bar{r}_{jj}^{\curlyvee} - 1 \right) + \bar{x}_{3jj}^{\curlyvee} \tan^{-1} \left(\frac{x_1}{\bar{x}_{3jj}^{\curlyvee}} \right)_{x_3=a_1}^{x_3=a_2} \right]_{x_1=-b}^{x_1=b} \\ &- \left[x_1 \log \bar{r}_{jj}^{\curlyvee} + \bar{x}_{3jj}^{\curlyvee} \tan^{-1} \left(\frac{x_1}{\bar{x}_{3jj}^{\curlyvee}} \right)_{x_1=-b}^{x_1=b} \right]_{x_3=a_1}^{x_3=a_2} = 0, \end{aligned} \tag{122}$$

$$\begin{aligned} I_{10} = &\left[x_1 \left(\log r_{jk}^{\curlyvee} - 1 \right) + x_{3jk}^{\curlyvee} \tan^{-1} \left(\frac{x_1}{x_{3jk}^{\curlyvee}} \right)_{x_3=a_1}^{x_3=a_2} \right]_{x_1=-b}^{x_1=b} \\ &- \left[x_1 \log r_{jk}^{\curlyvee} + x_{3jk}^{\curlyvee} \tan^{-1} \left(\frac{x_1}{x_{3jk}^{\curlyvee}} \right)_{x_1=-b}^{x_1=b} \right]_{x_3=a_1}^{x_3=a_2} = 0. \end{aligned} \tag{123}$$

Substituting the value of T from equation (97) into equation (107), and with the help of $S_3 = \sqrt{\frac{K_1^*}{K_3^*}} = S_3^{\curlyvee}$ and the integrals (112) and (113), one can derive the following:

$$M_{23} A_3 I_{11} + M_{23} \sum_{k=1}^{5} A_{3k} I_{12} + M_{23}^{\curlyvee} A_3^{\curlyvee} I_{13} + M_{23}^{\curlyvee} \sum_{k=1}^{4} A_{3k}^{\curlyvee} I_{14} = \frac{H_3}{\sqrt{\frac{K_3^*}{K_1^*}}}, \tag{124}$$

where

$$I_{11} = -\left[\tan^{-1}\left(\frac{x_1}{\bar{x}_{333}}\right)_{x_3=a_1}^{x_3=a_2}\right]_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{x_1}{\bar{x}_{333}}\right)_{x_1=-b}^{x_1=b}\right]_{x_3=a_1}^{x_3=h^-}$$
$$+ \left[\tan^{-1}\left(\frac{x_1}{\bar{x}_{333}}\right)_{x_1=-b}^{x_1=b}\right]_{x_3=h^+}^{x_3=a_2} = -2\pi, \tag{125}$$

$$I_{12} = -\left[\tan^{-1}\left(\frac{x_1}{x_{33k}}\right)_{x_3=a_1}^{x_3=a_2}\right]_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{x_1}{\bar{x}_{33k}}\right)_{x_1=-b}^{x_1=b}\right]_{x_3=a_1}^{x_3=a_2} = 0, \tag{126}$$

$$I_{13} = -\left[\tan^{-1}\left(\frac{x_1}{\bar{x}^{\Upsilon}_{333}}\right)_{x_3=a_1}^{x_3=a_2}\right]_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{x_1}{\bar{x}^{\Upsilon}_{333}}\right)_{x_1=-b}^{x_1=b}\right]_{x_3=a_1}^{x_3=h^-}$$
$$+ \left[\tan^{-1}\left(\frac{x_1}{\bar{x}^{\Upsilon}_{333}}\right)_{x_1=-b}^{x_1=b}\right]_{x_3=h^+}^{x_3=a_2} = -2\pi, \tag{127}$$

$$I_{14} = -\left[\tan^{-1}\left(\frac{x_1}{x^{\Upsilon}_{33k}}\right)_{x_3=a_1}^{x_3=a_2}\right]_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{x_1}{x^{\Upsilon}_{33k}}\right)_{x_1=-b}^{x_1=b}\right]_{x_3=a_1}^{x_3=a_2} = 0. \tag{128}$$

The following is obtained by utilizing equations (125), (126), (127), and (128) in Equation (124):

$$A_3 = -\frac{H_3}{2\pi(M_{23} + \vartheta_1 M^{\Upsilon}_{23})\sqrt{\frac{K_3^*}{K_1^*}}}. \tag{129}$$

With the assistance of $S_4 = \sqrt{\frac{D_1^*}{D_3^*}} = S_4^{\Upsilon}$ and the integrals (114) and (115), the value of P from equation (98) can be substituted into equation (108) to obtain

$$M_{34}A_4I_{15} + M_{34}\sum_{k=1}^{5} A_{4k}I_{16} + M^{\Upsilon}_{34}A^{\Upsilon}_4 I_{17} + M^{\Upsilon}_{34}\sum_{k=1}^{4} A^{\Upsilon}_{4k}I_{18} = \frac{H_4}{\sqrt{\frac{D_3^*}{D_1^*}}}, \tag{130}$$

where

$$I_{15} = -\left[\tan^{-1}\left(\frac{x_1}{\bar{x}_{344}}\right)\right]_{x_1=a_1}^{x_3=a_2}\Bigg|_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{x_1}{\bar{x}_{344}}\right)\right]_{x_1=-b}^{x_1=b}\Bigg|_{x_3=a_1}^{x_3=h^-}$$
$$+ \left[\tan^{-1}\left(\frac{x_1}{\bar{x}_{344}}\right)\right]_{x_1=-b}^{x_1=b}\Bigg|_{x_3=h^+}^{x_3=a_2} = -2\pi, \tag{131}$$

$$I_{16} = -\left[\tan^{-1}\left(\frac{x_1}{x_{34k}}\right)\right]_{x_3=a_1}^{x_3=a_2}\Bigg|_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{x_1}{x_{34k}}\right)\right]_{x_1=-b}^{x_1=b}\Bigg|_{x_3=a_1}^{x_3=a_2} = 0, \tag{132}$$

$$I_{17} = -\left[\tan^{-1}\left(\frac{x_1}{\bar{x}^{\Upsilon}_{344}}\right)\right]_{x_1=a_1}^{x_3=a_2}\Bigg|_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{x_1}{\bar{x}^{\Upsilon}_{344}}\right)\right]_{x_1=-b}^{x_1=b}\Bigg|_{x_3=a_1}^{x_3=h^-}$$
$$+ \left[\tan^{-1}\left(\frac{x_1}{\bar{x}^{\Upsilon}_{344}}\right)\right]_{x_1=-b}^{x_1=b}\Bigg|_{x_3=h^+}^{x_3=a_2} = -2\pi, \tag{133}$$

$$I_{18} = -\left[\tan^{-1}\left(\frac{x_1}{x^{\Upsilon}_{34k}}\right)\right]_{x_3=a_1}^{x_3=a_2}\Bigg|_{x_1=-b}^{x_1=b} + \left[\tan^{-1}\left(\frac{x_1}{x^{\Upsilon}_{34k}}\right)\right]_{x_1=-b}^{x_1=b}\Bigg|_{x_3=a_1}^{x_3=a_2} = 0. \tag{134}$$

Using equations (131), (132), (133), and (134) in equation (130), we arrive at the following:

$$A_4 = -\frac{H_4}{2\pi(M_{34} + \vartheta_2 M^{\Upsilon}_{34})\sqrt{\frac{D_3^*}{D_1^*}}}. \tag{135}$$

Substituting the value of N from equation (99) in equation (109) and using the integrals (116) and (117), we get

$$\sum_{j=1}^{5} M_{4j} A_j \wp_j + \sum_{j=1}^{4} \wp_j^{\Upsilon} \sum_{k=1}^{4} M^{\Upsilon}_{4j} A^{\Upsilon}_{jk} = 0, \tag{136}$$

where

$$\begin{aligned}\wp_j = -\frac{b^2\pi}{S_j} + \frac{S_j^2-1}{S_j}\Bigg[&2b(S_ja_2 - S_jh)\log\sqrt{b^2+(S_ja_2-S_jh)^2}\\ &- 2b(S_ja_1 - S_jh)\log\sqrt{b^2+(S_ja_1-S_jh)^2}\\ &+ ((S_ja_2-S_jh)^2-b^2)\tan^{-1}\frac{b}{S_ja_2-S_jh}\\ &- ((S_ja_1-S_jh)^2-b^2)\tan^{-1}\frac{b}{S_ja_1-S_jh} - 3b(S_ja_2-S_ja_1)\Bigg],\end{aligned} \tag{137}$$

$$\begin{aligned}\wp_j^{\curlyvee} = \frac{S_j^2-1}{S_j}\Bigg[&2b(S_ja_2+S_kh)\log\sqrt{b^2+(S_ja_2+S_kh)^2} + ((S_ja_2+S_kh)^2)\\ &- b^2\tan^{-1}\frac{b}{S_ja_2+S_kh} - 2b(S_ja_1+S_kh)\log\sqrt{b^2+(S_ja_1+S_kh)^2}\\ &- ((S_ja_1+S_kh)^2-b^2)\tan^{-1}\frac{b}{S_ja_1+S_kh} - 3b(S_ja_2-S_ja_1)\Bigg].\end{aligned} \tag{138}$$

At the surface $x_3 = 0$, equation (92) provides:

$$\begin{aligned}&x_{3j} = 0, \quad h_k = S_kh, \quad x_{3j}^{\curlyvee} = 0, \quad h_k^{\curlyvee} = S_k^{\curlyvee}h, \quad x_{3jk} = h_k,\\ &r_{jk} = \sqrt{x_1^2+h_k^2}, \quad \bar{x}_{3jk} = -h_k, \quad \bar{r}_{jk} = \sqrt{x_1^2+h_k^2},\\ &x_{3jk}^{\curlyvee} = h_k^{\curlyvee}, \quad r_{jk}^{\curlyvee} = \sqrt{x_1^2+h_k^{\curlyvee 2}}, \quad \bar{x}_{3jk}^{\curlyvee} = -h_k^{\curlyvee}, \quad \bar{r}_{jk}^{\curlyvee} = \sqrt{x_1^2+h_k^{\curlyvee 2}}.\end{aligned} \tag{139}$$

Using equation (139) and $S_3 = \sqrt{\frac{K_1^*}{K_3^*}}$ and $S_4 = \sqrt{\frac{D_1^*}{D_3^*}}$ as references, the values of t_{33}, t_{31}, T, P, and N from equations (100), (102), (97), (98), and (99) are inserted into equation (64) to obtain the following:

$$\check{M}_{1j}A_j + \sum_{k=1}^{5}\check{M}_{1k}A_{kj} = 0, \quad j = 1,2,3,4,5 \quad ,$$

$$\check{M}^{\curlyvee}_{1j}A^{\curlyvee}_j + \sum_{k=1}^{4}\check{M}^{\curlyvee}_{1k}A^{\curlyvee}_{kj} = 0, \quad j = 1,2,3,4, \tag{140}$$

$$S_j\check{M}_{1j}A_j - \sum_{k=1}^{5}S_k\check{M}_{1k}A_{kj} = 0, \quad j = 1,2,3,4,5,$$

$$S^{\curlyvee}_j\check{M}^{\curlyvee}_{1j}A^{\curlyvee}_j - \sum_{k=1}^{4}S^{\curlyvee}_k\check{M}^{\curlyvee}_{1k}A^{\curlyvee}_{kj} = 0, \quad j = 1,2,3,4, \tag{141}$$

$$A_3 - A_{33} = 0, \quad A_{3k} = 0, \quad k = 1,2,4,5 \quad ,$$

$$A^{\curlyvee}_3 - A^{\curlyvee}_{33} = 0, \quad A^{\curlyvee}_{3k} = 0, \quad k = 1,2,4, \tag{142}$$

$$A_4 - A_{44} = 0, \quad A_{4k} = 0, \quad k = 1,2,3,5 \quad ,$$

$$A^{\curlyvee}_4 - A^{\curlyvee}_{44} = 0, \quad A^{\curlyvee}_{4k} = 0, \quad k = 1,2,3, \tag{143}$$

$$M_{4j}A_j - \sum_{k=1}^{5}M_{4k}A_{kj} = 0, \quad j = 1,2,3,4,5. \tag{144}$$

The remaining twenty-eight constants $A_j(j = 1,2,5)$, $A_{jk}(j,k = 1,2,3,4)$, $A^{\curlyvee}_j(j = 1,2)$ and $A^{\curlyvee}_{jk}(j,k = 1,2,3)$ can be determined by applying the Cramer's rule procedure to twenty-eight equations, specifically equations (102), (105), (136), (140), (141), (142), (143), and (144).

7 Specific Situations

Case I: Photothermoelastic

Sub-case I:

In the absence of diffusion effects i.e. where $\curlyvee^p_1 = \curlyvee^p_3 = D^p_1 = D^p_3 = P = 0$ in equations (59)-(63), the corresponding expressions for photothermoelastic material (point heat source on the surface of semi-infinite medium) are expressed as

$$\begin{aligned} u_1 = &-\sum_{j=1}^{4}A_j\left[x_1\left(\log r_j - 1\right) + x_{3j}\tan^{-1}\frac{x_1}{x_{3j}}\right] \\ &-\sum_{j=1}^{3}A^{\curlyvee}_j\left[x_1\left(\log r^{\curlyvee}_j - 1\right) + x^{\curlyvee}_{3j}\tan^{-1}\frac{x_1}{x^{\curlyvee}_{3j}}\right], \end{aligned}$$

$$
\begin{aligned}
u_3 &= \sum_{j=1}^{4} S_j M_{1j} A_j \left[x_{3j} \left(\log r_j - 1 \right) - x_1 \tan^{-1} \frac{x_1}{x_{3j}} \right] \\
&\quad + \sum_{j=1}^{3} S_j^{\Upsilon} M_{1j}^{\Upsilon} A_j^{\Upsilon} \left[x_{3j}^{\Upsilon} \left(\log r_j^{\Upsilon} - 1 \right) - x_1 \tan^{-1} \frac{x_1}{x_{3j}^{\Upsilon}} \right], \\
T &= M_{23} A_3 \log r_3 + M_{23}^{\Upsilon} A_3^{\Upsilon} \log r_3^{\Upsilon}, \\
N &= \sum_{j=1}^{4} M_{3j} A_j \left[\frac{1}{2} \left(x_{3j}^2 - x_1^2 \right) \left(\log r_j - \frac{3}{2} \right) - x_1 x_{3j} \tan^{-1} \frac{x_1}{x_{3j}} \right], \\
t_{33} &= \sum_{j=1}^{4} \check{M}_{1j} A_j \log r_j + \sum_{j=1}^{3} \check{M}_{1j}^{\Upsilon} A_j^{\Upsilon} \log r_j^{\Upsilon}, \\
t_{11} &= -\sum_{j=1}^{4} S_j^2 \check{M}_{1j} A_j \log r_j - \sum_{j=1}^{3} S_j^{\Upsilon 2} \check{M}_{1j}^{\Upsilon} A_j^{\Upsilon} \log r_j^{\Upsilon}, \\
t_{31} &= -\sum_{j=1}^{4} S_j \check{M}_{1j} A_j \tan^{-1} \frac{x_1}{x_{3j}} - \sum_{j=1}^{3} S_j^{\Upsilon} \check{M}_{1j}^{\Upsilon} A_j^{\Upsilon} \tan^{-1} \frac{x_1}{x_{3j}^{\Upsilon}}.
\end{aligned} \tag{145}
$$

Sub-case II:

In the absence of diffusion effects, taking $\gamma_1^p = \gamma_3^p = D_1^p = D_3^p = P = 0$ in equations (95)-(102), the expressions for a photothermoelastic material (point heat source in the interior of a semi-infinite medium) are as follows:

$$
\begin{aligned}
u_1 = &-\sum_{j=1}^{4} A_j \left[x_1 \left(\log \bar{r}_{jj} - 1\right) + \bar{x}_{3jj} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\
&- \sum_{j=1}^{4}\sum_{k=1}^{4} A_{jk} \left[x_1 \left(\log r_{jk} - 1\right) + x_{3jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right] \\
&- \sum_{j=1}^{3} A_j^{\Upsilon} \left[x_1 \left(\log \bar{r}_{jj}^{\Upsilon} - 1\right) + \bar{x}_{3jj}^{\Upsilon} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}^{\Upsilon}}\right)\right] \\
&- \sum_{j=1}^{3}\sum_{k=1}^{3} A_{jk}^{\Upsilon} \left[x_1 \left(\log r_{jk}^{\Upsilon} - 1\right) + x_{3jk}^{\Upsilon} \tan^{-1}\left(\frac{x_1}{x_{3jk}^{\Upsilon}}\right)\right]. \\
u_3 = &\sum_{j=1}^{4} S_j M_{1j} A_j \left[\bar{x}_{3jj} \left(\log \bar{r}_{jj} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\
&+ \sum_{j=1}^{4}\sum_{k=1}^{4} S_j M_{1j} A_{jk} \left[x_{3jk} \left(\log r_{jk} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right] \\
&+ \sum_{j=1}^{3} S_j^{\Upsilon} M_{1j}^{\Upsilon} A_j^{\Upsilon} \left[\bar{x}_{3jj}^{\Upsilon} \left(\log \bar{r}_{jj}^{\Upsilon} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}^{\Upsilon}}\right)\right] \\
&+ \sum_{j=1}^{3}\sum_{k=1}^{3} S_j^{\Upsilon} M_{1j}^{\Upsilon} A_{jk}^{\Upsilon} \left[x_{3jk}^{\Upsilon} \left(\log r_{jk}^{\Upsilon} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{x_{3jk}^{\Upsilon}}\right)\right]. \\
T = &M_{23} A_3 \log \bar{r}_{33} + M_{23} \sum_{k=1}^{4} A_{3k} \log r_{3k} \\
&+ M_{23}^{\Upsilon} A_3^{\Upsilon} \log \bar{r}_{33}^{\Upsilon} + M_{23}^{\Upsilon} \sum_{k=1}^{3} A_{3k}^{\Upsilon} \log r_{3k}^{\Upsilon}. \\
N = &\sum_{j=1}^{4} M_{3j} A_j \left[\frac{1}{2}(\bar{x}_{3jj}^2 - x_1^2)\left(\log \bar{r}_{jj} - \frac{3}{2}\right) - x_1 \bar{x}_{3jj} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\
&+ \sum_{j=1}^{4}\sum_{k=1}^{4} M_{3j} A_{jk} \left[\frac{1}{2}(x_{3jk}^2 - x_1^2)\left(\log r_{jk} - \frac{3}{2}\right) - x_1 x_{3jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right]. \\
t_{33} = &\sum_{j=1}^{4} \check{M}_{1j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{4}\sum_{k=1}^{4} \check{M}_{1j} A_{jk} \log r_{jk} + \sum_{j=1}^{3} \check{M}_{1j}^{\Upsilon} A_j^{\Upsilon} \log \bar{r}_{jj}^{\Upsilon} \\
&+ \sum_{j=1}^{3}\sum_{k=1}^{3} \check{M}_{1j}^{\Upsilon} A_{jk}^{\Upsilon} \log r_{jk}^{\Upsilon}. \\
t_{11} = &-\sum_{j=1}^{4} S_j^2 \check{M}_{1j} A_j \log \bar{r}_{jj} - \sum_{j=1}^{4}\sum_{k=1}^{4} S_j^2 \check{M}_{1j} A_{jk} \log r_{jk} \\
&- \sum_{j=1}^{3} (S_j^{\Upsilon})^2 \check{M}_{1j}^{\Upsilon} A_j^{\Upsilon} \log \bar{r}_{jj}^{\Upsilon} - \sum_{j=1}^{3}\sum_{k=1}^{3} (S_j^{\Upsilon})^2 \check{M}_{1j}^{\Upsilon} A_{jk}^{\Upsilon} \log r_{jk}^{\Upsilon}. \\
t_{31} = &-\sum_{j=1}^{4} S_j \check{M}_{1j} A_j \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right) - \sum_{j=1}^{4}\sum_{k=1}^{4} S_j \check{M}_{1j} A_{jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right) \\
&- \sum_{j=1}^{3} S_j^{\Upsilon} \check{M}_{1j}^{\Upsilon} A_j^{\Upsilon} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}^{\Upsilon}}\right) - \sum_{j=1}^{3}\sum_{k=1}^{3} S_j^{\Upsilon} \check{M}_{1j}^{\Upsilon} A_{jk}^{\Upsilon} \tan^{-1}\left(\frac{x_1}{x_{3jk}^{\Upsilon}}\right).
\end{aligned}
\tag{146}
$$

Case II: Thermoelastic

Sub-case I:

Taking $\gamma_1^p = \gamma_3^p = D_1^p = D_3^p = P = \gamma_1^n = \gamma_3^n = D_1^* = D_3^* = N = 0$ in equations (59)-(63), the corresponding expressions for thermoelastic material (point heat source on the surface of a semi-infinite medium) are obtained as:

$$u_1 = -\sum_{j=1}^{3} A_j \left[x_1 (\log r_j - 1) + x_{3j} \tan^{-1} \left(\frac{x_1}{x_{3j}} \right) \right],$$
$$u_3 = \sum_{j=1}^{3} S_j M_{1j} A_j \left[x_{3j} (\log r_j - 1) - x_1 \tan^{-1} \left(\frac{x_1}{x_{3j}} \right) \right],$$
$$T = M_{23} A_3 \log r_3, \ t_{33} = \sum_{j=1}^{3} \check{M}_{1j} A_j \log r_j,$$
$$t_{11} = -\sum_{j=1}^{3} S_j^2 \check{M}_{1j} A_j \log r_j, \ t_{31} = -\sum_{j=1}^{3} S_j \check{M}_{1j} A_j \tan^{-1} \left(\frac{x_1}{x_{3j}} \right). \qquad (147)$$

The results are comparable to those of Kumar and Chawla [10].

Sub-case II:

Taking $\gamma_1^p = \gamma_3^p = D_1^p = D_3^p = P = \gamma_1^n = \gamma_3^n = D_1^* = D_3^* = N = 0$ in equations (95)-(102), we obtain the corresponding expressions for thermoelastic material (point heat source in the interior of a semi-infinite medium) as:

$$
\begin{aligned}
u_1 &= -\sum_{j=1}^{3} A_j \left[x_1 \left(\log \bar{r}_{jj} - 1\right) + \bar{x}_{3jj} \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\
&\quad - \sum_{j=1}^{3}\sum_{k=1}^{3} A_{jk} \left[x_1 \left(\log r_{jk} - 1\right) + x_{3jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right], \\
u_3 &= \sum_{j=1}^{3} S_j M_{1j} A_j \left[\bar{x}_{3jj} \left(\log \bar{r}_{jj} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right)\right] \\
&\quad + \sum_{j=1}^{3}\sum_{k=1}^{3} S_j M_{1j} A_{jk} \left[x_{3jk} \left(\log r_{jk} - 1\right) - x_1 \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right)\right], \\
T &= M_{23} A_3 \log \bar{r}_{33} + M_{23} \sum_{k=1}^{3} A_{3k} \log r_{3k}, \\
t_{33} &= \sum_{j=1}^{3} \check{M}_{1j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{3}\sum_{k=1}^{3} \check{M}_{1j} A_{jk} \log r_{jk}, \\
t_{11} &= -\sum_{j=1}^{3} S_j^2 \check{M}_{1j} A_j \log \bar{r}_{jj} - \sum_{j=1}^{3}\sum_{k=1}^{3} S_j^2 \check{M}_{1j} A_{jk} \log r_{jk}, \\
t_{31} &= -\sum_{j=1}^{3} S_j \check{M}_{1j} A_j \tan^{-1}\left(\frac{x_1}{\bar{x}_{3jj}}\right) - \sum_{j=1}^{3}\sum_{k=1}^{3} S_j \check{M}_{1j} A_{jk} \tan^{-1}\left(\frac{x_1}{x_{3jk}}\right).
\end{aligned}
\tag{148}
$$

The results obtained tally with those obtained by Kumar and Chawla [11].

8 Numerical results and discussion

For the numerical computations, we implement the orthotropic Silicon (Si) material constants (Kumar et al., [14], Kumar and Kansal[13]) as

$$C_{11} = 19.45N/m^2,\ C_{13} = 6.41N/m^2,\ C_{33} = 16.57N/m^2,\ C_{55} = 7.96N/m^2,$$
$$\alpha_1^t = 2.33\times10^{-5}K^{-1}, \alpha_3^t = 2.48\times10^{-5}K^{-1},\quad \alpha_1^c = 2.65\times10^{-4}m^3Kg^{-1},$$
$$\alpha_3^c = 2.83\times10^{-4}m^3Kg^{-1},\ \gamma_1 = -0.029715m^3, \gamma_3 = -0.02714m^3,$$
$$\rho = 0.2328Kg/m^3,\quad D_1^p = 1.85Kgm^{-3}s,\ D_3^p = .95Kgm^{-3}s,\ E_g = 1.11eV,$$
$$K_1^* = 170Ns^{-1}K^{-1},\ K_3^* = 165Ns^{-1}K^{-1},\quad D_1^*4.0m^2/s,\ D_3^* = 3.5m^2/s,$$
$$C_e = 710J/KgK, L = 1m,\ n_o = 10^{20}m^{-3},\quad a = 2.9\times10^4m^2s^{-2}K^{-1},$$
$$b = .9m^5s^{-2}Kg^{-1},\ \tau = 5s,\ \zeta = 0.5m^{-3}K^{-1}.$$

Numerical computation has been accomplished via MATLAB (R2014a) software at the plane $x_3 = 2$. The Figures 2(a)-3(a) and Figures 4(a)-5(a) show how the normal stress (t_{33}), temperature distribution (T), and carrier density (N) changes with distance (x_1) for (i) point heat source H_1=1, H_2=0 (ii) diffusion source H_1=0, H_2=1 applied at the origin of semi-infinite medium in the MGTPWD model and an orthotropic photothermoelastic medium without diffusion based on the Moore-Gibson-Thompson (MGTPWTD) model. Figure 3(b) and 5(b) shows the variation of chemical potential P with x_1 for b=1.9 and b=2.9 on the plane $x_3 = 2$ for (i) point heat source H_1=1, H_2=0 (ii) chemical potential source H_1=0, H_2=1.

Figures 6(a)-7(a) and 8(a)-9(a) show how the same field variables change with distance x_1 for (i) point heat source H_3=1, H_4=0 (ii) chemical potential source H_3=0, H_4=1 applied at point (0, h) in a semi-infinite medium for the MGTPWD model and the MGTPWTD model. Figure 7(b)-9(b) depicts the variation of chemical potential P changes at b=1.9 and b=2.9 in case of (i) point heat source H_3=1, H_4=0 (ii) chemical potential source H_3=0, H_4=1 at the point (0, h).

Deliberation of the fundamental solution

Heat source

Figure 2(a) shows how the normal stress t_{33} changes and behaves with respect to x_1. The MGTPWD model has more changes in the average stress t_{33} between $0 \le x_1 \le 3$. The function is stable and tends to zero in the range $3 \le x_1$ for both models. Figure 2(b) shows how the temperature distribution T changes as x_1 changes. In both the MGTPWD and MGTPWTD models, T behaves almost the same, with only small differences in how big they are. Figure 3(a) shows how the carrier density N changes as x_1 changes. The carrier density N has a low number at the beginning of both models, and it rises steadily with distance. The value of N stays higher for the MGTPWTD model compared to the MGTPWD model. Figure 3(b) shows how the chemical potential P changes as x_1 changes. For b=1.9 and b=2.9, the behaviour and changes in the shapes that correspond to P are the same. As b goes from 1.9 to 2.9, the curves show an upward trend, and P stays large at 1.9.

Chemical potential source

Figure 4(a) shows how t_{33} changes as x_1 changes. At first, the MGTPWTD model's normal stress t_{33} is less stable than the MGTPWD model's. The value of t_{33} stays high for the MGTPWD model compared to the MGTPWTD model. Figure 4(b) shows how temperature T changes as x_1 changes. For both models, the value of T goes up as the distance grows. Its magnitude is still bigger for the MGTPWTD model than for the MGTPWD model. Figure 5(a) shows how the carrier density N changes as x_1 changes. At the start, the carrier density N is at its lowest point. It then starts to rise with distance for both the models. The value of N stays bigger for the MGTPWD model than for the MGTPWTD model. Figure 5(b) shows how the chemical potential P changes as x_1 changes. Both the curves correspond to exhibit increasing behavior for b=1.9 and b=2.9. The size of P is bigger when b=2.9 than when b=1.9.

Deliberation of the Greens function

Heat source

Figure 6(a) shows how t_{33} changes as x_1 changes. In the range $1 \leq x_1 \leq 4$, t_{33} exhibits fluctuating behavior for MGTPWTD model whereas its magnitude decreases for MGTPWD model. Under the MGTPWTD model, the magnitude of t_{33} exhibits a reduction and tend to zero in the range $4 \leq x_1 \leq 10$, however under the MGTPWD model, the magnitude of t_{33} follows an increasing trend. Figure 6(b) shows how the T changes as x_1 changes. The magnitude of T on the MGTPWD model has less variation than the MGTPWTD model. For the MGTPWTD model, T stays at a high level. Figure 7(a) shows how N changes as x_1 changes. At first, the value of N is very small, but it grows as x_1 progresses for both models, but for the MGTPWTD model N has a bigger value. The change P with respect to x_1 is shown in Figure 7(b). It is true that the magnitude of P goes up from b=1.9 to b=2.9, with b=1.9 having the larger magnitude.

Chemical potential source

Figure 8(a) shows how the normal stress t_{33} changes as x_1 changes. For the MGTPWTD model, the normal stress t_{33} oscillates in the range $1 \leq x_1 \leq 4$, but for the MGTPWD model, it gets smaller. When $4 \leq x_1 \leq 10$, the difference between the magnitudes of t_{33} grows as the distance grows.Figure 8(b) shows how temperature T changes as x_1 changes. The MGTPWD model has more changes in the magnitude of T than the MGTPWTD model, but the magnitude of T stays high for the MGTPWD model. Figure 9(a) shows how the carrier density N changes as x_1 changes. The MGTPWD model's N magnitude is higher than the MGTPWTD model's. As the distance grows, the difference in magnitude decreases until it finally equalizes. Figure 9(b) shows how the chemical potential P changes as x_1 changes. It is true that for both b=1.9 and b=2.9, the chemical potential P increases as distance move farther away, and the difference between the magnitudes of P get bigger.

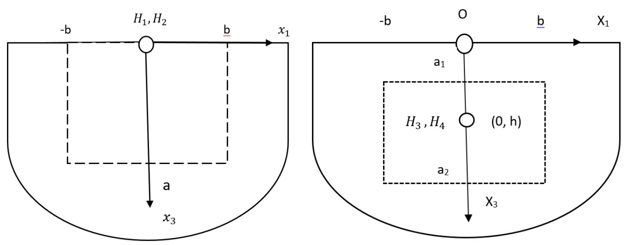
In conclusion

The orthotropic MGTPWD model has been used to derive the fundamental solution and Green's function for a two-dimensional problem. Nine newly introduced harmonic functions are used to construct the fundamental solution and Green's function for a stable point heat source on the surface and in the interior of a semi-infinite orthotropic photothermoelastic material as a result of the general solution's effects. The elementary functions are used to express the components of displacement, stress, temperature change diffusion, and carrier density distribution. From numerical computed results following are observed.

In case of fundamental solution, t_{33} oscillates for both the assumed models for both heat source and chemical potential source. t_{33} attains maximum magnitude for MGTPWD model and minimum for MGTPWTD model. Regarding the heat source, the magnitude of T is larger for MGTPWD model in comparison to MGTPWD model. In case of chemical potential source, the magnitude of T follow a reverse pattern. For the two postulated models, the magnitude of N rises monotonically. In case of heat source magnitude of N is higher for MGTPWD model while in case of chemical potential source its magnitude is higher for MGTPWTD model. When a heat source is present, the magnitude of chemical potential P is larger at b = 1.9 than it is at b = 2.9 but when a chemical potential source is present, the magnitude of P follow opposite trend.

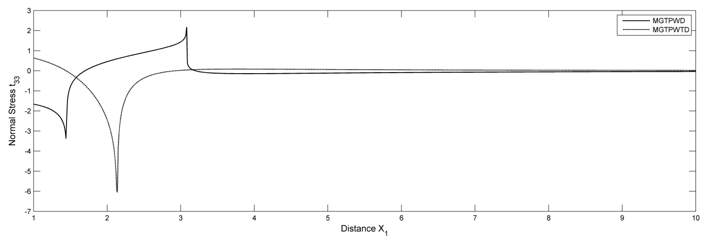
In case of Greens function, for both the heat source and the chemical potential source, the MGTPWTD model exhibits greater oscillating in t_{33} when compared to the MGTPWD model. For both the imagined models, magnitude of T rises monotonically for both heat source and chemical potential source but in a reversal trend.When analyzing a heat source, the difference in magnitudes of N for the two assumed models grows with increasing distance, however when analyzing a chemical potential source, the difference in magnitudes diminishes. In contrast to b = 2.9, the chemical potential P magnitude for b = 1.9 is larger for the chemical potential source as well as the heat source.

The proposed model is distinct in its confirmation and physical meaning is clear. It is appropriate, practically useful and deliver additional assesses to estimate how a material behaviour acts in the real world.it is also concluded that all the field quantities are sufficiently restricted to thermal and diffusion parameters. Physical views presented in this article are useful for the design and scientific domains.

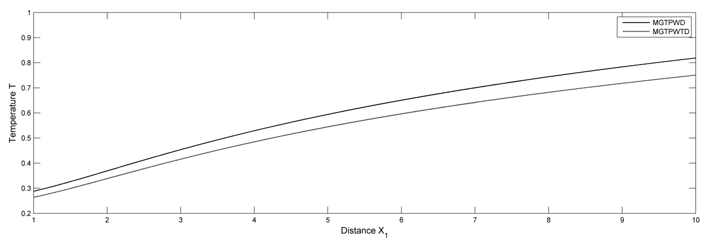


(a) (b)

Figure 1: Geometry of the problem

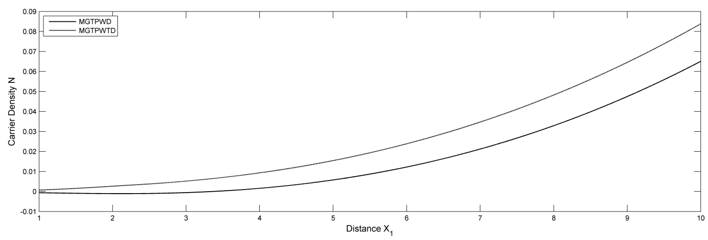


(a) Variation of normal stress t_{33} with respect to x_1

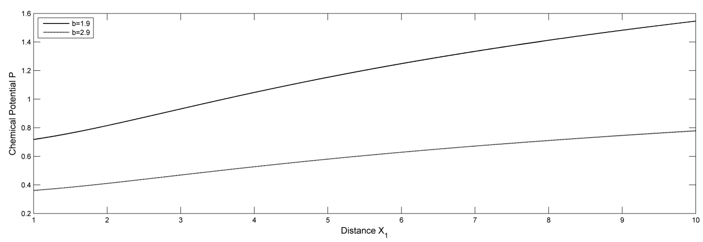


(b) Variation of temperature distribution T with respect to x_1

Figure 2:

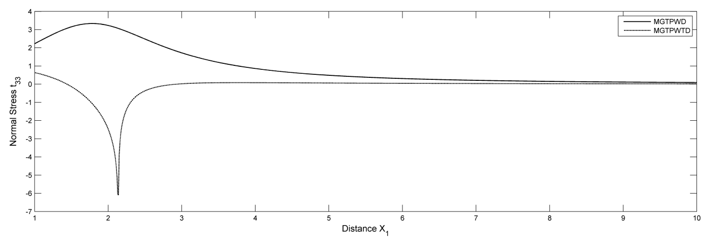


(a) Variation of carrier density N with respect to x_1

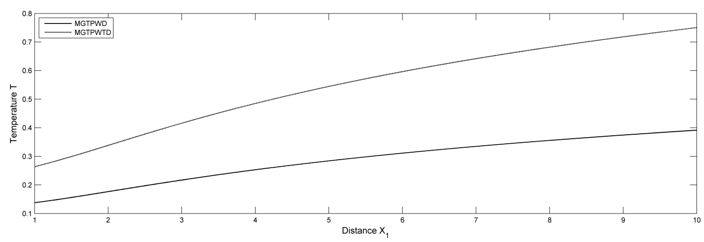


(b) Variation of chemical potential P with respect to x_1

Figure 3:

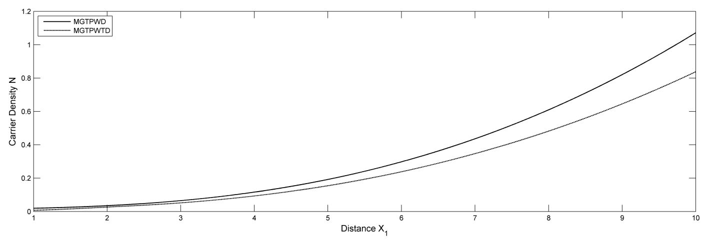


(a) Variation of normal stress t_{33} with respect to x_1

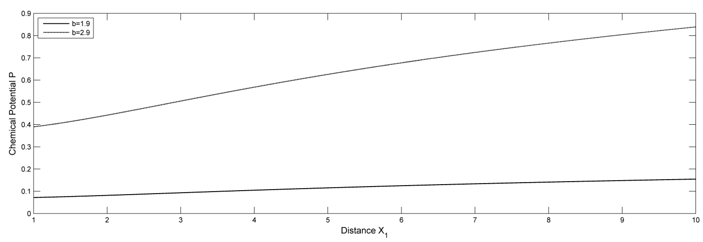


(b) Variation of temperature distribution T with respect to x_1

Figure 4:

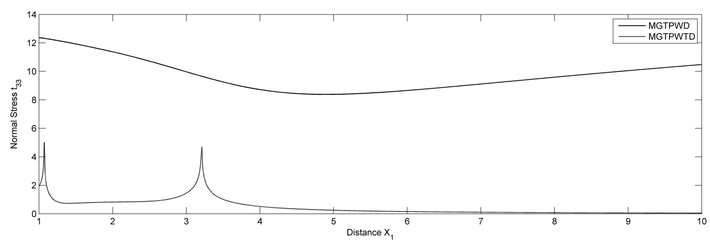


(a) Variation of carrier density N with respect to x_1

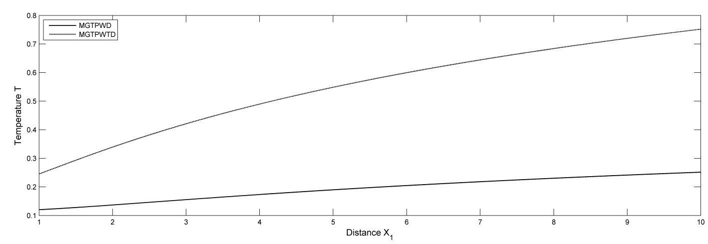


(b) Variation of chemical potential P with respect to x_1

Figure 5:

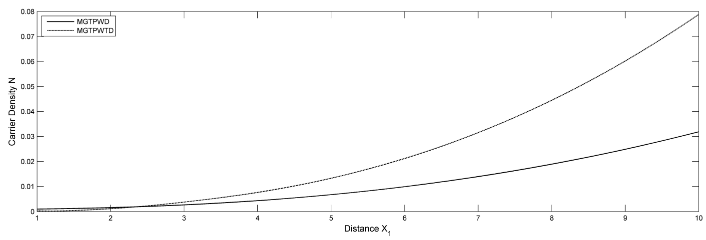


(a) Variation of normal stress t_{33} with respect to x_1

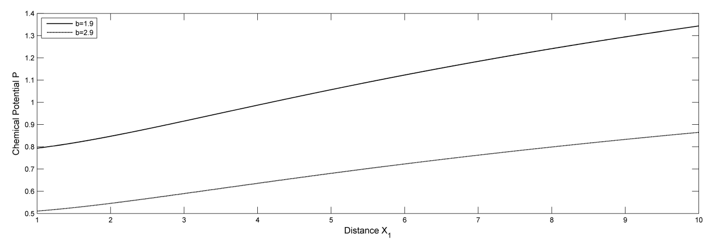


(b) Variation of temperature distribution T with respect to x_1

Figure 6:

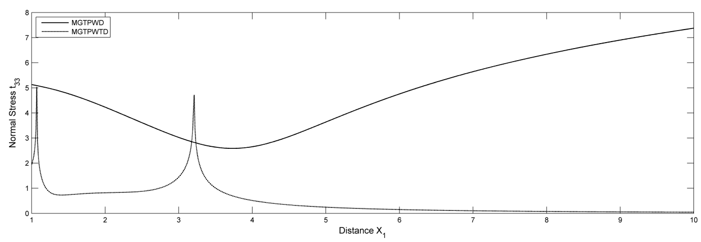


(a) Variation of carrier density N with respect to x_1

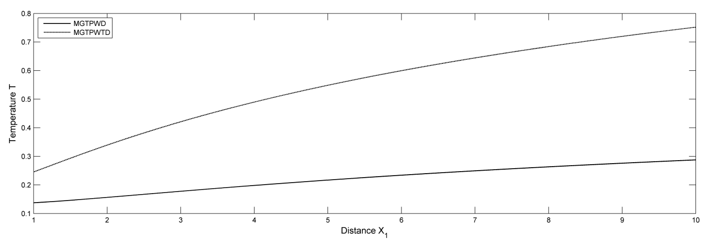


(b) Variation of chemical potential P with respect to x_1

Figure 7:

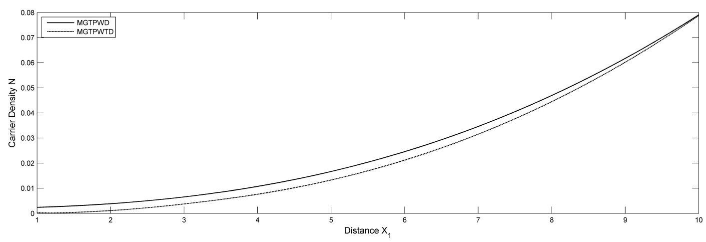


(a) Variation of normal stress t_{33} with respect to x_1

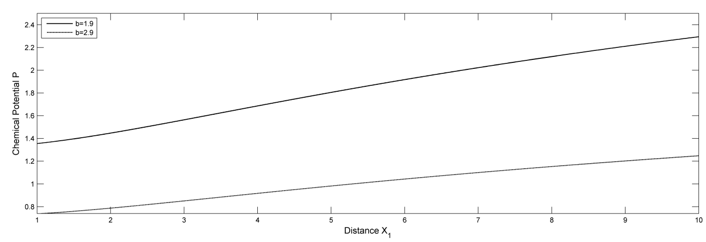


(b) Variation of temperature distribution T with respect to x_1

Figure 8:



(a) Variation of carrier density N with respect to x_1



(b) Variation of chemical potential P with respect to x_1

Figure 9:

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Saurav Sharma,
Department of Industrial Engineering,
University of Houston,
17900 Cambridge Street, 7-2G,
Houston, Texas-77054, USA.
Email: sauravkuk@gmail.com