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A mathematical note about a FLRW universe embedded in a universe without time

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Abstract

In this paper we propose to study a six-dimensional Friedmann-Lemaître-Robertson-Walker (FLRW) universe without time. The way we study this timeless universe is the classic one: we highlight future oriented time-like loops and closed chains of future oriented time-like curves. Inside this six-dimensional FLRW universe it is embedded a four-dimensional classical FLRW universe.

1 Introduction

In this article we obtain an example of a FriedmannLemaîtreRobertson Walker (FLRW) six-dimensional expanding universe that is timeless according to Gödel's meaning (see [5]).

There is a series of articles published by W.G. Boskoff (see [1], [2], [3]) related to timeless universes. In [3], a family of timeless universes is constructed. This model has led us to consider the possibility of a six-dimensional timeless universe containing a four-dimensional FLRW physical universe, where the time exists.

The four-dimensional spacetime is studied in special and general relativity, and theoretical models for the physical expanding universe are described (see [4], [7]), while the importance of the FLRW metrics to model our universe is highlighted (see [6]).

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The concept of a timeless six-dimensional universe housing our expanding four-dimensional reality represents a different approach from the conventional cosmology (see [8]). While the mathematical framework presented here demonstrates that it is theoretically feasible, one possible implication is that the time of our universe is arising from a higher-dimensional structure.

2 Main result: A six-dimensional timeless FLRW universe containing a four-dimensional FLRW physical universe

We first consider a six-dimensional metric on a set. Using an appropriate change of coordinates, we can transfer the metric to another set and obtain a new six-dimensional metric satisfying Einstein's field equations, describing a FLRW expanding universe.

Consider the metric

$$ds^2 = e^{x^3}(dx^0)^2 - e^{2\sqrt{(x^4)^2+(x^5)^2}} \cdot \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right] + \\ +(dx^4)^2 + (dx^5)^2 \tag{1}$$

on $M_1 = \mathbb{R}^6$ and, in order to simplify the computations, we apply to this metric the following change of coordinates

$$F_1 : \begin{cases} x^0 = t, \\ x^1 = x \\ x^2 = y \\ x^3 = z \\ x^4 = r\cos\theta \\ x^5 = r\sin\theta, \end{cases}$$

where $t, x, y, z, \theta \in \mathbb{R},\ r \in (0, \infty)$.

Therefore, we obtain the following metric connected with (1) through the above transformation

$$d\tilde{s}^2 = e^z dt^2 - e^{2r}(dx^2 + dy^2 + dz^2) + dr^2 + r^2 d\theta^2 \tag{2}$$

on the set $M_2 = \mathbb{R}^4 \times (0, \infty) \times \mathbb{R}$ described by the coordinates (t, x, y, z, r, θ).

Denoting $(t, x, y, z, r, \theta) := (y^0, y^1, y^2, y^3, y^4, y^5)$, the metric (2) becomes

$$d\tilde{s}^2 = e^{y^3}(dy^0)^2 - e^{2y^4} \cdot \left[(dy^1)^2 + (dy^2)^2 + (dy^3)^2\right] + (dy^4)^2 + (y^4)^2(dy^5)^2.$$

So, the nonzero components g_{ij} of this metric tensor are

$$g_{00} = e^{y^3},\ g_{11} = g_{22} = g_{33} = -e^{2y^4},\ g_{44} = 1,\ g_{55} = (y^4)^2.$$

We compute the Christoffel symbols for metric (2) and they are

$$\Gamma^0_{03} = \Gamma^0_{30} = \frac{1}{2};\ \Gamma^3_{00} = \frac{1}{2}e^{y^3-2y^4};$$

$$\Gamma^1_{14} = \Gamma^1_{41} = \Gamma^2_{24} = \Gamma^2_{42} = \Gamma^3_{34} = \Gamma^3_{43} = 1;$$

$$\Gamma^4_{11} = \Gamma^4_{22} = \Gamma^4_{33} = e^{2y^4};$$

$$\Gamma^4_{55} = -y^4;$$

$$\Gamma^5_{45} = \Gamma^5_{54} = \frac{1}{y^4}.$$

All the other Christoffel symbols are zero.
Therefore, the components R_{ij} of the Ricci tensor for metric (2) are

$$R_{00} = \frac{1}{4}e^{y^3-2y^4};$$

$$R_{11} = R_{22} = 3e^{2y^4} + \frac{e^{2y^4}}{y^4};$$

$$R_{33} = 3e^{2y^4} + \frac{e^{2y^4}}{y^4} - \frac{1}{4};$$

$$R_{34} = R_{43} = \frac{1}{2};$$

$$R_{44} = -3;$$

$$R_{55} = -3y^4.$$

All the other components R_{ij} of the Ricci tensor are zero.
So, we obtain that the Ricci scalar curvature is

$$R = \frac{e^{-2y^4}}{2} - \frac{6}{y^4} - 12.$$

Therefore, the Einstein's field equations

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi G T_{ij},$$

are verified for the stress-energy tensor $T_{ij} = \frac{1}{8\pi G}\tilde{T}_{ij}$, where G is the gravitational constant and

$$\tilde{T}_{00} = \frac{3e^{y^3}(1+2y^4)}{y^4}$$

$$\tilde{T}_{11} = \tilde{T}_{22} = \frac{1}{4} - \frac{e^{2y^4}(2+3y^4)}{y^4}$$

$$\tilde{T}_{33} = -\frac{e^{2y^4}(2+3y^4)}{y^4}$$

$$\tilde{T}_{44} = \frac{12(y^4+1) - y^4 e^{-2y^4}}{4y^4}$$

$$\tilde{T}_{55} = \frac{(y^4)^2(24 - e^{-2y^4})}{4}$$

$$\tilde{T}_{34} = \tilde{T}_{43} = \frac{1}{2}$$

and the rest of $\tilde{T}_{ij}$ are zero.

The coordinates (x, y, z) from the second metric completely determine the coordinates (x^1, x^2, x^3) from the first metric. The coordinate t from the second metric completely determines the coordinate x^0 from the first metric. The coordinates (r, θ) from the second metric completely determine the coordinates (x^4, x^5) from the first metric.

The nature of this coordinates, time type or space type, will be settled defining the orientation of M_2. We will define the orientation such that x, y, z are space type coordinates and t, r, θ are time type coordinates in M_2 (and, corresponding to them, x^1, x^2, x^3 are space type coordinates and x^0, x^4, x^5 are time type coordinates in M_1). The orientation will be defined following Gödel's model.

The vectors in the tangent spaces of M_2 can be split in three types of vectors: time-like, space-like and light-like vectors.

A vector $u = (u^0, u^1, u^2, u^3, u^4, u^5)$ of a tangent space of M_2 is a time-like vector if $d\tilde{s}^2(u, u) = g_{ij}u^i u^j > 0$, it is a space-like vector if $d\tilde{s}^2(u, u) < 0$ or it is a light-like vector if $d\tilde{s}^2(u, u) = 0$.

We see that considering the vectors

$$e_0 := (1, 0, 0, 0, 0, 0), e_4 := (0, 0, 0, 0, 1, 0), e_5 := (0, 0, 0, 0, 0, 1),$$

we have that $d\tilde{s}^2(e_0, e_0) = e^z > 0$, $d\tilde{s}^2(e_4, e_4) = 1 > 0$, $d\tilde{s}^2(e_5, e_5) = r^2 > 0$, so the vectors e_0, e_4, e_5 are time-like and considering the vectors

$$e_1 := (0, 1, 0, 0, 0, 0), e_2 := (0, 0, 1, 0, 0, 0), e_3 := (0, 0, 0, 1, 0, 0),$$

we have that $d\tilde{s}^2(e_1, e_1) = -e^{2r} < 0$, $d\tilde{s}^2(e_2, e_2) = -e^{2r} < 0$, $d\tilde{s}^2(e_3, e_3) = -e^{2r} < 0$, so the vectors e_1, e_2, e_3 are space-like.

Therefore the coordinates t, r, θ are time type and the coordinates x, y, z are space type in M_2 (and, consequently, x^0, x^4, x^5 are time type coordinates and x^1, x^2, x^3 are space type coordinates in M_1).

The vector $\tilde{e} = (e^0, e^1, e^2, e^3, e^4, e^5) = (1, 0, 0, 0, 1, 1)$ is time-like because $d\tilde{s}^2(\tilde{e}, \tilde{e}) = e^z + 1 + r^2 > 0$. We choose this vector to establish the future oriented time-like vectors.

We say that a time-like vector $u = (u^0, u^1, u^2, u^3, u^4, u^5)$ is future oriented if $d\tilde{s}^2(\tilde{e}, u) = g_{ij}e^i u^j > 0$ and it is past oriented if $d\tilde{s}^2(\tilde{e}, u) = g_{ij}e^i u^j < 0$. If a time-like vector u is future oriented, then the vector $-u$ is a time-like past oriented vector.

We say that a curve $c(s)$ in M_2 is a time-like future oriented curve if all its tangent vectors $\dot{c}(s)$ are time-like future oriented vectors. In this case, the curve $F(c(s))$ in M_1 is also considered a future oriented time-like curve.

We can now formulate the following result.

Theorem 1. *The set M_1 with the metric (1) allows future oriented time-like loops.*

Proof. Consider in M_2 the curve $c(s) := (0, R_1 \cos s, R_1 \sin s, 0, \ln R_2, s)$, where $s \in [0, 2\pi]$. This curve is modeling the movement of a point in M_2 (which moves in space and also in time).

The corresponding moving point

$$F(c(s)) := (0, R_1 \cos s, R_1 \sin s, 0, \ln R_2 \cdot \cos s, \ln R_2 \cdot \sin s)$$

in M_1 has also a movement in space (for the second and the third coordinates) and a movement in time (for the fifth and the sixth coordinates).

We assume that $R_2 > 1$, so $\frac{\ln R_2}{R_2} > 0$ and we can choose R_1 such that $0 < R_1 < \frac{\ln R_2}{R_2}$. Therefore $R_1^2 < \frac{\ln^2 R_2}{R_2^2}$, so $R_1^2 R_2^2 < \ln^2 R_2$ and we obtain that $-R_1^2 R_2^2 + \ln^2 R_2 > 0$.

The velocity vector of $c(s)$ is

$$\dot{c}(s) = (0, -R_1 \sin s, R_1 \cos s, 0, 0, 1) = (v^0, v^1, v^2, v^3, v^4, v^5),$$

therefore

$$d\tilde{s}^2(\dot{c}(s), \dot{c}(s)) = g_{ij}v^i v^j = g_{11}(v^1)^2 + g_{22}(v^2)^2 + g_{55}(v^5)^2 =$$

$$= -e^{2\ln R_2}(R_1^2 \sin^2 s + R_1^2 \cos^2 s) + \ln^2 R_2 = -R_1^2 R_2^2 + \ln^2 R_2 > 0,$$

hence $\dot{c}(s)$ is a time-like vector.

Moreover,

$$d\tilde{s}^2(\tilde{e}, \dot{c}(s)) = g_{ij}e^i v^j = g_{55}e^5 v^5 = \ln^2 R_2 > 0,$$

and consequently $\dot{c}(s)$ is a future oriented time-like vector.

The points $c(0) = (0, R_1, 0, 0, \ln R_2, 0)$ and $c(2\pi) = (0, R_1, 0, 0, \ln R_2, 2\pi)$ are connected in M_2 by the future oriented time-like curve $c(s)$. But they have the same image in M_1, that is the point $A(0, R_1, 0, 0, \ln R_2, 0) \in M_1$, because $F(c(0)) = F(c(2\pi)) = (0, R_1, 0, 0, \ln R_2, 0)$. We obtain that the image in M_1 of the curve $c(s)$, which is also a future oriented time-like curve, starts from $A = F(c(0))$ and it is returning to the same point $A = F(c(2\pi))$ when $s \in [0, 2\pi]$, so it is a future oriented time-like loop in M_1. □

Consequence 1. *The coordinates x^4 and x^5 in M_1 with the metric (1) are not proper time coordinates.*

Proof. We use the time-like loop $F(c(s)) \subset M_1$, constructed in the proof of Theorem 1. The curve $F(c(s)), s \in [0, 2\pi]$ is a future oriented time-like curve startig with the point $A = F(c(0))$, so each point of the time-like curve $F(c(s))$ (with $s \in (0, 2\pi]$) is an event in the future of the initial event A. Because $A = F(c(2\pi))$, we obtain that the point A is in the future of A, that is impossible. The time coordinates involved in this situation are x^4 and x^5, therefore they are not proper time coordinates. □

Consider now the following

Remark 1. *Let $t_1, t_2 \in \mathbb{R}$ and choose $R_1, R_2 > 0$ such that $R_2 > 1$ and $0 < R_1 < \frac{\ln R_2}{R_2}$ (like in the proof of theorem 1). For R_2 big enough we have that both inequalities*

$$\left(\frac{t_2 - t_1}{2\pi}\right)^2 - R_1^2 R_2^2 + \ln^2 R_2 > 0 \tag{3}$$

$$\frac{t_2 - t_1}{2\pi} + \ln^2 R_2 > 0 \tag{4}$$

hold.

We can now formulate

Theorem 2. *Consider $t_1, t_2 \in \mathbb{R}$ and $R_1 > 0, R_2 > 1$ satisfying relations (3) and (4).*

Any two points A_{t_1} and A_{t_2}, which are images in M_1 respectively of the points $(t_1, R_1, 0, 0, \ln R_2, 0)$ and $(t_2, R_1, 0, 0, \ln R_2, 2\pi)$ in M_2 can be connected in M_1 by a future oriented time-like curve such that A_{t_2} is in the future of A_{t_1}.

Proof. Let B_{t_1} and B_{t_2} be the two points in M_2 having the coordinates $(t_1, R_1, 0, 0, \ln R_2, 0)$ and $(t_2, R_1, 0, 0, \ln R_2, 2\pi)$, respectively.

Consider now the curve

$$\gamma(s) = \left(t_1 + \frac{t_2 - t_1}{2\pi} \cdot s, R_1 \cos s, R_1 \sin s, 0, \ln R_2, s\right),$$

connecting in M_2 the points $B_{t_1} = \gamma(0)$ and $B_{t_2} = \gamma(2\pi)$.

The tangent vector of $\gamma(s)$ is

$$\dot{\gamma}(s) = \left(\frac{t_2 - t_1}{2\pi}, -R_1 \sin s, R_1 \cos s, 0, 0, 1\right) = (v^0, v^1, v^2, v^3, v^4, v^5).$$

We have that along the curve $\gamma(s)$ the tangent vector satisfies

$$d\tilde{s}^2(\dot{\gamma}(s), \dot{\gamma}(s)) = g_{ij} v^i v^j = g_{00}(v^0)^2 + g_{11}(v^1)^2 + g_{22}(v^2)^2 + g_{55}(v^5)^2 =$$

$$= e^0 \cdot \left(\frac{t_2 - t_1}{2\pi}\right)^2 - e^{2 \ln R_2}(R_1^2 \sin^2 s + R_1^2 \cos^2 s) + \ln^2 R_2 =$$

$$= \left(\frac{t_2 - t_1}{2\pi}\right)^2 - R_1^2 R_2^2 + \ln^2 R_2 > 0,$$

and, moreover,

$$d\tilde{s}^2(\tilde{e}(s), \dot{\gamma}(s)) = g_{ij} e^i v^j = g_{00} e^0 v^0 + g_{55} e^5 v^5 =$$

$$= \frac{t_2 - t_1}{2\pi} + \ln^2 R_2 > 0,$$

therefore $\gamma(s)$ is a future oriented time-like curve in M_2.

The image of $\gamma(s)$ in M_1 is

$$F(\gamma(s)) = \left(t_1 + \frac{t_2 - t_1}{2\pi} \cdot s, R_1 \cos s, R_1 \sin s, 0, \ln R_2 \cdot \cos s, \ln R_2 \cdot \sin s\right),$$

$s \in [0, 2\pi]$, connecting in M_1 the points

$$A_{t_1} = F(B_{t_1}) = F(\gamma(0)) = (t_1, R_1, 0, 0, \ln R_2, 0)$$

and

$$A_{t_2} = F(B_{t_2}) = F(\gamma(2\pi)) = (t_2, R_1, 0, 0, \ln R_2, 0).$$

The curve $F(\gamma(s))$ is a future oriented time-like curve in M_1, because it is the image of the future oriented time-like curve $\gamma(s)$ in M_1, therefore the event A_{t_2} is in the future of A_{t_1}. $\square$

Using Theorem 2, we can prove the following result

Theorem 3. *The set M_1 with the metric (1) allows closed chains of future oriented time-like curves.*

Proof. Consider $t_1, t_2, t_3 \in \mathbb{R}$ and let $R_1, R_2 > 0$ such that $R_2 > 1$ and $0 < R_1 < \frac{\ln R_2}{R_2}$. For R_2 big enough we have that the inequalities

$$\left(\frac{t_j - t_i}{2\pi}\right)^2 - R_1^2 R_2^2 + \ln^2 R_2 > 0$$
$$\frac{t_j - t_i}{2\pi} + \ln^2 R_2 > 0$$

hold, for any $(i, j) \in \{(1, 2), (2, 3), (3, 1)\}$.

Using the idea in the proof of Theorem 2, we consider in M_2 the following future oriented time-like curves:

$$\alpha(s) = \left(t_1 + \frac{t_2 - t_1}{2\pi} \cdot s, R_1 \cos s, R_1 \sin s, 0, \ln R_2, s\right),$$
$$\beta(s) = \left(t_2 + \frac{t_3 - t_2}{2\pi} \cdot s, R_1 \cos s, R_1 \sin s, 0, \ln R_2, s\right),$$
$$\gamma(s) = \left(t_3 + \frac{t_1 - t_3}{2\pi} \cdot s, R_1 \cos s, R_1 \sin s, 0, \ln R_2, s\right),$$

with $s \in [0, 2\pi]$.

Their images in M_1 are the curves

$$F(\alpha(s)) = \left(t_1 + \frac{t_2 - t_1}{2\pi} \cdot s, R_1 \cos s, R_1 \sin s, 0, \ln R_2 \cdot \cos s, \ln R_2 \cdot \sin s\right),$$
$$F(\beta(s)) = \left(t_2 + \frac{t_3 - t_2}{2\pi} \cdot s, R_1 \cos s, R_1 \sin s, 0, \ln R_2 \cdot \cos s, \ln R_2 \cdot \sin s\right),$$
$$F(\gamma(s)) = \left(t_3 + \frac{t_1 - t_3}{2\pi} \cdot s, R_1 \cos s, R_1 \sin s, 0, \ln R_2 \cdot \cos s, \ln R_2 \cdot \sin s\right),$$

with $s \in [0, 2\pi]$, which are future oriented time-like curves in M_1.

We see that

$$F(\alpha(0)) = F(\gamma(2\pi)) = (t_1, R_1, 0, 0, \ln R_2, 0),$$
$$F(\beta(0)) = F(\alpha(2\pi)) = (t_2, R_1, 0, 0, \ln R_2, 0),$$
$$F(\gamma(0)) = F(\beta(2\pi)) = (t_3, R_1, 0, 0, \ln R_2, 0).$$

Therefore this curves are connecting in M_1 the points:
$F(\alpha(0))$ and $F(\alpha(2\pi)) = F(\beta(0))$, $F(\beta(0))$ and $F(\beta(2\pi)) = F(\gamma(0))$, $F(\gamma(0))$ and $F(\gamma(2\pi)) = F(\alpha(0))$, therefore we obtain in M_1 a closed chain of future oriented time-like curves (the concatenation of the curves $\alpha(s), \beta(s), \gamma(s)$, with $s \in [0, 2\pi]$). □

We can now formulate

Consequence 2. *The coordinates x^0, x^4 and x^5 in M_1 with the metric (1) are not proper time coordinates.*

Proof. We see from the proof of Theorem 3 that the event $E_2 = F(\alpha(2\pi))$ is in the future of the event $E_1 = F(\alpha(0))$, the event $E_3 = F(\beta(2\pi))$ is in the future of $E_2 = F(\beta(0))$ and the event $E_1 = F(\gamma(2\pi))$ is in the future of $E_3 = F(\gamma(0))$. We obtain that the event E_1 is in its own future, which is not possible for proper time coordinates.

The coordinates involved in this situation are x^0, x^4 and x^5, so they cannot be proper time coordinates. □

Remark 2. *We have that M_1 with the metric (1) is modeling a Friedmann LemaîtreRobertsonWalker (FLRW) expanding universe. Using the above results, we obtain that M_1 is a timeless FLRW universe.*

If we restrict the set $M_1 = \mathbb{R}^6$ for $x^0 = x^5 = 0$, the metric (1) becomes a four-dimensional FLRW physical expanding universe, where the time exists:

$$ds^2 = (dx^4)^2 - e^{2|x^4|} \cdot \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right]. \tag{5}$$

In this universe representing the region $0 \times \mathbb{R}^4 \times 0$ of $M_1 = \mathbb{R}^6$, the coordinate x^4 is a proper time coordinate and the spatial component of the metric is time dependent.

This special region is in fact a physical four-dimensional spacetime of FLRW type, where the classical time exists and the expansion of the space is given by a scale factor depending on this physical time. The scale factor of this FLRM universe has a positive second derivative, therefore the expansion of this universe is accelerating.

Similarly, we obtain a four-dimensional FLRW-universe in the region of $M_1 = \mathbb{R}^6$ where $x^0 = x^4 = 0$:

$$ds^2 = (dx^5)^2 - e^{2|x^5|} \cdot \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right]. \tag{6}$$

Here, x^5 is a proper time coordinate.

3 Conclusions

In this paper, we constructed a six-dimensional timeless FLRW universe, demonstrating that it contains a four-dimensional FLRW spacetime which may be our physical universe. This theoretical model offers a new perspective on the nature of time and the underlying structure of the physical universe, providing the idea that our universe may be embedded in a higher-dimensional universe.

We proved that a six-dimensional FLRW-like metric can provide a consistent framework for a four-dimensional FLRW physical expanding universe. This model challenges traditional cosmology by suggesting that the observable dynamics may be projections of a higher-dimensional geometry. The absence of the proper time coordinates in the six-dimensional universe raises more questions about the origin of temporal flow in our universe. The possibility that our universe is a region of a larger timeless reality invites us to reconsider the conventional cosmological theories and provides a useful starting point for future research.

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