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# Parameter estimation for a SEIRS model with COVID-19 data of Türkiye

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**Abstract**

In this paper, the unknown parameters of a SEIRS mathematical model for the dynamics of COVID-19 are estimated by the least squares approach using data of Trkiye. In the considered model, the infective group is divided into two classes consisting of diagnosed and undiagnosed individuals. Since the data for undiagnosed infective individuals in the community is unknown, three different scenarios are proposed. The numerical solutions of the model using the estimated parameter values and the actual data are demonstrated with graphs.

## 1 Introduction

COVID-19 is the disease caused by a new coronavirus called SARS-CoV-2 which is first defined on December 31, 2019, in Wuhan, People's Republic of China ([7]). The World Health Organization (WHO) declared COVID-19 outbreak a pandemic on March 11, 2020.

COVID-19 pandemic has taken its place as an important inflection point in world's history. Besides being a disease that causes fatal health problems, COVID-19 has also caused great damage to the psychology of societies and the economy in the global sense. The results of this destruction appeared independent of the economic and sociocultural differences between countries. Today, it is accepted by many scientists that the control strategies against

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COVID-19 such as vaccination, use of masks, social distancing and complying with hygiene rules, etc. play the most important role in preventing this epidemic.

Mathematical models are very important tools to explain the dynamics of epidemic diseases ([5], [16], [10]). Recently, many mathematical model studies have been carried out on COVID-19 to predict and explain the mechanism and transmission of the disease ([22], [15], [11], [25], [21], [24], [1]). Some of these studies on COVID-19 have particularly focused on the estimation of the unknown parameters in introduced models. In [17], Mehra et al have considered SIR and SQAIR models for the COVID-19 with the real data of South Korea. They estimated the unknown parameters; transmission rate, recovery rate, and mortality rate in the model. A simple SIR model was considered and parameters were estimated by Bagal et al in India during lockdown periods ([2]). In [3], SIR and SIAS models were studied to forecast the evolution of the COVID-19 with the real data of Brazil by the authors. Bentout et al. consider a SEIR epidemic model for COVID-19 and they predicted the basic reproduction number $R_0 = 4.1$ in [4]. Mekonen, Habtemicheal, and Balcha considered a SEIRD model and estimated the unknown parameters using the combination of least squares and Bayesian estimation in [18]. They showed that the basic reproduction number is dependent also in the contamination of environment, besides the infection and recovery rates.

In a recent study, a novel COVID-19 model was proposed by Demirci ([8]). The basic reproduction number was obtained and local stability analysis of the disease-free equilibrium was given. In this study, the same model given in [8] has been considered without vaccination and some unknown parameters in the model have been estimated by using COVID-19 data of Türkiye, that is reported to World Health Organization by the Ministry of Health of the Republic of Türkiye ([26]). In addition three different data scenarios have been proposed for the undiagnosed infective individuals in the community and results are discussed.

## 2 The Model

In this study, we consider the model given in [8] that consists of five compartments: $S(t), E(t), I_1(t), I_2(t)$ and $R(t)$. $S(t)$ is composed of the susceptible individuals in the population. The infectious group is divided into three classes that are exposed ($E(t)$), infective who are diagnosed with COVID-19 ($I_1(t)$) and infective who are not diagnosed (both symptomatic and asymptomatic) ($I_2(t)$). The individuals in the exposed group are infected but due to the incubation period of the virus they are not contagious until the last two days of this period ([5]). The diagnosis rate of these individuals is also quite

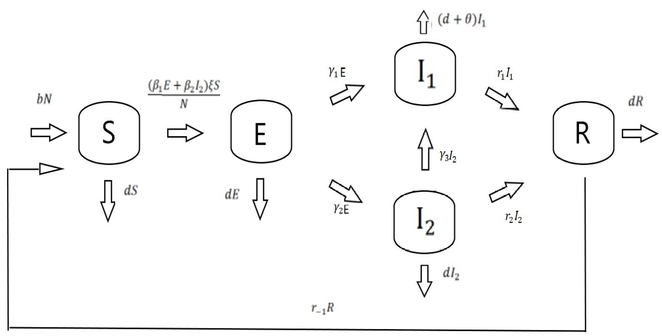



Figure 1: Flow chart for the model (1a)-(1e)

low even though they are tested. Since the disease is not distributed in the country equally, in this paper, the total population $N(t)$ is assumed to denote the number of individuals who have been tested [17]. We also used fixed birth and death rates ($b$ and $d$, respectively) in this study. The flowchart of the model is given in Figure 1. The system that represents the considered model is

$$\frac{dS}{dt} = bN - \frac{\beta_1 E + \beta_2 I_2}{N}\xi S - dS + r_{-1}R, \tag{1a}$$

$$\frac{dE}{dt} = \frac{\beta_1 E + \beta_2 I_2}{N}\xi S - (\gamma_1 + \gamma_2 + d)E, \tag{1b}$$

$$\frac{dI_1}{dt} = \gamma_1 E + \gamma_3 I_2 - (d + \theta + r_1)I_1, \tag{1c}$$

$$\frac{dI_2}{dt} = \gamma_2 E - (\gamma_3 + r_1 + d)I_2, \tag{1d}$$

$$\frac{dR}{dt} = r_1 I_1 + r_2 I_2 - (d + r_{-1})R. \tag{1e}$$

with non-negative initial conditions $S(0) = S_0$, $E(0) = E_0$, $I_1(0) = I_{10}$, $I_2(0) = I_{20}$, $R(0) = R_0$. Here, $\beta_1$ and $\beta_2$ denote the transmission rates due to the interaction of the class $S$ with the classes $E$ and $I_2$, respectively. $\sqrt{\xi}$ represents the rate of protected individuals by social isolation. $\gamma_1$ and $\gamma_3$ are rates of being diagnosed for individuals who are in the classes $E$ and $I_2$, respectively and $\gamma_2$ is the rate of being undiagnosed for the class $E$. $r_1$, $r_2$ and $r_{-1}$ are waning immunity rate, recovery rate for $I_1$ and $I_2$, respectively.

## 3 Data Collection and Method

In this study, parameters of the model are estimated based on COVID-19 data for Türkiye between December 12, 2020 and January 25, 2021. In Türkiye, vaccination against COVID-19 has started on 14 January, 2021 but it took some more time for the vaccination to become massively effective. Therefore, for the considered 45 days of period, parameter estimation is performed using a model without vaccination term. Data of daily COVID-19 cases are collected from the reports of COVID-19 information platform of the Republic of Türkiye, Ministry of Health, given to WHO [26]. The number of active cases at time $t$ which corresponds to the variable $I_1(t)$ is obtained by subtracting the number of deaths and the number of recovered individuals from the daily number of cases announced by the Ministry, starting from the first day of pandemic. Data on COVID-19 prevalence that include the undiagnosed cases are extremely limited in Türkiye. One of the main motivations of this study is to obtain information about the number of undiagnosed cases in public according to CoVID-19 data of Türkiye. There are many studies in the literature on the number of the undiagnosed individuals who are infected with SARS-CoV-2 virus ([14]) and most of them introduce their prediction or hypothesis on this number with the help of statistical methods ([6], [23]). In a recent study, the number of the undiagnosed individuals is given as 5 times of diagnosed cases ([12]). Hamadeh et al used a mathematical modeling approach to estimate the unobserved incidence in Canada and they estimated the proportion of the infected population that remained undiagnosed in Canadian provinces at a certain time interval ([9]). According to this study, the estimated total number of infections ranged from 2 to 6 times the diognosed cases. In [13] Ruiyu et al have shown that the suppression of the infectiousness of the undocumented cases in model simulations reduces the total number of documented cases and the overall spread of SARS-CoV-2.

In this context, we consider three different scenarios to generate $I_2$ data. Three different intervals [1,3], [4,6], [7,9] are considered and $I_2$ is generated as the product of the $I_1$ data and the randomly generated number series from these intervals using uniform distribution. For instance, for the first scenario, at time $t_i$, $I_2$ is defined as:

$$I_2(t_i) = rnd[1,3] \times I_1(t_i).$$

For the other scenarios data are obtained similarly. Generated $I_2$ data for three scenarios are shown in Fig. 2.

Data set for exposed class is generated using the formula:

$$E(t) = I_1(t+6) + I_2(t+6),\ t = 0, 1, \ldots, 44.$$

We note that the incubation period for COVID-19 is assumed to be 7 days. It is known that 80%-90% of those who recover from disease gain immunity for 6 months. Hence, data for the recovered (immune) class, R(t), for the first scenario is constructed using the following formula:

$$R(t) = (1 - \frac{1}{180})(R(t-1) + 0.9V(t)(1 + rnd[1,3])). \quad (2)$$

Here, $V(t)$ is the number of daily recovered individuals at time $t$ and, $R(t-1)$ is the number of immune individuals at time $t-1$ ([26]). We note that, the random effect on the recovery class, $rnd[1,3]$, in Equation 2 is replaced by $rnd[4,6]$ and $rnd[7,9]$ for the scenarios 2 and 3, respectively.

As mentioned before, here $N$ is considered as the total test number in Türkiye ([17]) and the value of $N$ is fixed to 28.648.193, the total number of people who have been tested on the last day (45th day) of the data([26]). Generated data sets for the first, second and third scenarios are given in Table 1, 2 and 3, respectively.

In this paper, to estimate the values of unknown parameters, we minimize an objective function which represents the errors in least squares sense, relatively. The objective function $\mathcal{S}$ that is minimized is formulated as:

$$\mathcal{S} = \sum_{j=1}^{m} \frac{\sum_{i=1}^{n} \|x_j(t_i) - x_{ji}\|^2}{\sum_{i=1}^{n} \|\hat{x}_{ji} - x_{ji}\|^2},$$

where $n$ is the number of actual data (in our problem n=45) and $m$ is the number of independent variables in the model which is 5. Here, $x_1(t_i)$, $x_2(t_i)$, $x_3(t_i)$, $x_4(t_i)$ and $x_5(t_i)$ correspond to values $S(t_i)$, $E(t_i)$, $I_1(t_i)$, $I_2(t_i)$, $R(t_i)$

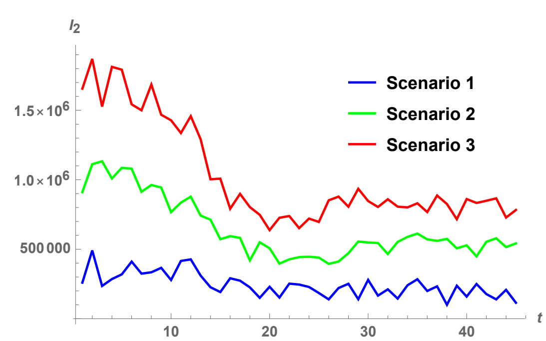



Figure 2: $I_2$ data for three scenarios

| time | $S$ | $E$ | $I_1$ | $I_2$ | $R$ |
|---|---|---|---|---|---|
| 0. | 26987795 | 535321 | 212045 | 259516 | 653516 |
| 1. | 26734236 | 541408 | 216531 | 491221 | 664797 |
| 2. | 26947774 | 572309 | 217755 | 235632 | 674723 |
| 3. | 26981520 | 469209 | 220375 | 285313 | 691776 |
| 4. | 26809806 | 593179 | 219931 | 319016 | 706261 |
| 5. | 26713386 | 589374 | 216709 | 410825 | 717899 |
| 6. | 26900153 | 456733 | 210928 | 324393 | 755986 |
| 7. | 26957083 | 354561 | 207366 | 334042 | 795141 |
| 8. | 26893217 | 311445 | 206218 | 366091 | 871222 |
| 9. | 26812119 | 403393 | 190648 | 278561 | 963472 |
| 10. | 26653183 | 378900 | 177543 | 415636 | 1022931 |
| 11. | 26623799 | 325310 | 162442 | 426932 | 1109710 |
| 12. | 26730270 | 245204 | 146305 | 310428 | 1215986 |
| 13. | 26705262 | 316965 | 128081 | 226480 | 1271405 |
| 14. | 26787039 | 237224 | 119715 | 191730 | 1312485 |
| 15. | 26537148 | 336646 | 112470 | 290923 | 1371006 |
| 16. | 26467369 | 329597 | 105207 | 273693 | 1472327 |
| 17. | 26452783 | 315812 | 99755 | 225555 | 1554288 |
| 18. | 26484910 | 277741 | 95001 | 150203 | 1640338 |
| 19. | 26396475 | 236860 | 87121 | 229844 | 1697893 |
| 20. | 26348810 | 322339 | 85002 | 152222 | 1739820 |
| 21. | 26168353 | 354271 | 84308 | 252338 | 1788923 |
| 22. | 26240576 | 244663 | 83890 | 245707 | 1833357 |
| 23. | 26073909 | 384629 | 87492 | 228320 | 1873843 |
| 24. | 26196524 | 271154 | 92884 | 184857 | 1902774 |
| 25. | 26125339 | 316074 | 97821 | 139039 | 1969920 |
| 26. | 26071672 | 249157 | 101587 | 220752 | 2005025 |
| 27. | 25921604 | 345133 | 102986 | 251285 | 2027185 |
| 28. | 25974412 | 387375 | 104440 | 140223 | 2041743 |
| 29. | 25903286 | 302306 | 105299 | 279330 | 2057972 |
| 30. | 25981034 | 333097 | 105044 | 166110 | 2062908 |
| 31. | 26060839 | 198308 | 104669 | 211405 | 2072972 |
| 32. | 25977737 | 335047 | 104587 | 144570 | 2086252 |
| 33. | 25946054 | 256808 | 104368 | 240765 | 2100198 |
| 34. | 25808870 | 347497 | 103404 | 283971 | 2104451 |
| 35. | 25963417 | 274855 | 102781 | 199525 | 2107615 |
| 36. | 25960590 | 236611 | 100240 | 232857 | 2117895 |
| 37. | 26016456 | 304575 | 98033 | 100275 | 2128854 |
| 38. | 25962298 | 210471 | 97466 | 237581 | 2140377 |
| 39. | 25894318 | 347610 | 97810 | 158998 | 2149457 |
| 40. | 25876270 | 258238 | 97833 | 249664 | 2166188 |
| 41. | 25917529 | 283644 | 97633 | 177222 | 2172165 |
| 42. | 25885491 | 341092 | 97534 | 139077 | 2184999 |
| 43. | 25952988 | 196595 | 96811 | 207764 | 2194035 |
| 44. | 25955326 | 274891 | 95634 | 114837 | 2207505 |

Table 1: Actual Data for Scenario 1

| time | S | E | I1 | I2 | R |
|---|---|---|---|---|---|
| 0. | 24914697 | 1124155 | 212045 | 911183 | 1486113 |
| 1. | 24552713 | 1169777 | 216531 | 1112021 | 1597151 |
| 2. | 24432800 | 1150427 | 217755 | 1132899 | 1714312 |
| 3. | 24572932 | 957577 | 220375 | 1009289 | 1888020 |
| 4. | 24264193 | 1013234 | 219931 | 1085842 | 2064993 |
| 5. | 24093967 | 1040671 | 216709 | 1079573 | 2217273 |
| 6. | 24258941 | 888597 | 210928 | 913227 | 2376500 |
| 7. | 24160139 | 840796 | 207366 | 962411 | 2477481 |
| 8. | 24209408 | 691710 | 206218 | 944209 | 2596648 |
| 9. | 24186303 | 706456 | 190648 | 766929 | 2797857 |
| 10. | 23964427 | 687049 | 177543 | 835691 | 2983483 |
| 11. | 23905098 | 519424 | 162442 | 878229 | 3183000 |
| 12. | 23736413 | 644950 | 146305 | 742292 | 3378233 |
| 13. | 23663055 | 594183 | 128081 | 712715 | 3550159 |
| 14. | 23819956 | 481341 | 119715 | 571995 | 3655186 |
| 15. | 23700424 | 511581 | 112470 | 593986 | 3729732 |
| 16. | 23625689 | 527085 | 105207 | 581842 | 3808370 |
| 17. | 23695316 | 533450 | 99755 | 419669 | 3900003 |
| 18. | 23483533 | 532942 | 95001 | 549949 | 3986768 |
| 19. | 23498470 | 492377 | 87121 | 507062 | 4063163 |
| 20. | 23524732 | 513131 | 85002 | 396339 | 4128989 |
| 21. | 23404160 | 574170 | 84308 | 427273 | 4158282 |
| 22. | 23271751 | 659928 | 83890 | 443195 | 4189429 |
| 23. | 23241505 | 653942 | 87492 | 445958 | 4219296 |
| 24. | 23222001 | 649559 | 92884 | 440058 | 4243691 |
| 25. | 23326455 | 570305 | 97821 | 394556 | 4259056 |
| 26. | 23205520 | 657403 | 101587 | 411544 | 4272139 |
| 27. | 23070435 | 693197 | 102986 | 471184 | 4310391 |
| 28. | 22936307 | 716008 | 104440 | 555488 | 4335950 |
| 29. | 22964840 | 674036 | 105299 | 548643 | 4355375 |
| 30. | 22960966 | 660393 | 105044 | 544515 | 4377275 |
| 31. | 23007506 | 672617 | 104669 | 465636 | 4397765 |
| 32. | 22954357 | 603814 | 104587 | 552816 | 4432619 |
| 33. | 22864715 | 625834 | 104368 | 588829 | 4464447 |
| 34. | 22897080 | 546545 | 103404 | 612604 | 4488560 |
| 35. | 22809276 | 651106 | 102781 | 571255 | 4513775 |
| 36. | 22767600 | 676294 | 100240 | 560153 | 4543906 |
| 37. | 22802254 | 612211 | 98033 | 574584 | 4561111 |
| 38. | 22832111 | 637735 | 97466 | 506348 | 4574533 |
| 39. | 22801009 | 645686 | 97810 | 528024 | 4575664 |
| 40. | 23033153 | 490895 | 97833 | 448712 | 4577600 |
| 41. | 22929096 | 483506 | 97633 | 553473 | 4584485 |
| 42. | 22916196 | 460281 | 97534 | 578760 | 4595422 |
| 43. | 22938193 | 491185 | 96811 | 515400 | 4606604 |
| 44. | 22949552 | 449992 | 95634 | 542101 | 4610914 |

Table 2: Actual Data for Scenario 2

| time | S | E | I1 | I2 | R |
|---|---|---|---|---|---|
| 0. | 23514353 | 1710895 | 212045 | 1655727 | 1555173 |
| 1. | 23017394 | 1892669 | 216531 | 1869918 | 1651681 |
| 2. | 23419599 | 1674028 | 217755 | 1526761 | 1810050 |
| 3. | 23087247 | 1618647 | 220375 | 1812789 | 1909135 |
| 4. | 23060865 | 1513732 | 219931 | 1792206 | 2061459 |
| 5. | 23052098 | 1620098 | 216709 | 1542886 | 2216402 |
| 6. | 23146334 | 1438120 | 210928 | 1499967 | 2352844 |
| 7. | 23163813 | 1131703 | 207366 | 1685303 | 2460008 |
| 8. | 23301074 | 1127689 | 206218 | 1467810 | 2545402 |
| 9. | 23342807 | 904081 | 190648 | 1427999 | 2782658 |
| 10. | 23225717 | 1003540 | 177543 | 1336189 | 2905204 |
| 11. | 22986775 | 903001 | 162442 | 1457656 | 3138319 |
| 12. | 23034146 | 842692 | 146305 | 1291815 | 3333235 |
| 13. | 23281844 | 724631 | 128081 | 1003622 | 3510015 |
| 14. | 23178408 | 811670 | 119715 | 1007974 | 3530426 |
| 15. | 23301368 | 823226 | 112470 | 791611 | 3619518 |
| 16. | 23168466 | 735663 | 105207 | 898333 | 3740524 |
| 17. | 23106428 | 808154 | 99755 | 803246 | 3830610 |
| 18. | 23082165 | 789371 | 95001 | 747691 | 3933965 |
| 19. | 22973934 | 949997 | 87121 | 637510 | 3999631 |
| 20. | 22804371 | 980284 | 85002 | 726668 | 4051868 |
| 21. | 22840799 | 908913 | 84308 | 738918 | 4075255 |
| 22. | 22761283 | 1039385 | 83890 | 651773 | 4111862 |
| 23. | 22746191 | 952225 | 87492 | 720662 | 4141623 |
| 24. | 22793587 | 909263 | 92884 | 696487 | 4155972 |
| 25. | 22546757 | 964317 | 97821 | 852176 | 4187122 |
| 26. | 22549857 | 910891 | 101587 | 878697 | 4207161 |
| 27. | 22601097 | 905479 | 102986 | 805927 | 4232704 |
| 28. | 22435407 | 934051 | 104440 | 934945 | 4239350 |
| 29. | 22558472 | 870275 | 105299 | 846926 | 4267221 |
| 30. | 22461631 | 986870 | 105044 | 804219 | 4290429 |
| 31. | 22431554 | 923410 | 104669 | 859648 | 4328912 |
| 32. | 22569471 | 813480 | 104587 | 806304 | 4354351 |
| 33. | 22398204 | 958968 | 104368 | 801111 | 4385542 |
| 34. | 22380757 | 930915 | 103404 | 830647 | 4402470 |
| 35. | 22413241 | 946506 | 102781 | 767494 | 4418171 |
| 36. | 22269419 | 963683 | 100240 | 886630 | 4428221 |
| 37. | 22455435 | 824815 | 98033 | 825377 | 4444533 |
| 38. | 22506052 | 878246 | 97466 | 716014 | 4450415 |
| 39. | 22412909 | 809267 | 97810 | 861158 | 4467049 |
| 40. | 22356071 | 874823 | 97833 | 833082 | 4486384 |
| 41. | 22348810 | 870091 | 97633 | 848873 | 4482786 |
| 42. | 22303533 | 881644 | 97534 | 866149 | 4499333 |
| 43. | 22477626 | 839729 | 96811 | 728004 | 4506023 |
| 44. | 22495419 | 750146 | 95634 | 782612 | 4524382 |

Table 3: Actual Data for Scenario 3

which are predicted by the model at time $t_i$, respectively. Similarly, $x_{1i}$, $x_{2i}$, $x_{3i}$, $x_{4i}$ and $x_{5i}$ correspond to actual data $S_i$, $E_i$, $I_{1i}$, $I_{2i}$, $R_i$ at time $t_i$, respectively. Finally, $\hat{x}_{ji}$, $j = 1, \ldots, m$, $i = 1, \ldots, n$ are the mean values of the actual data in each compartment.

## 4 Model Parameter Estimation

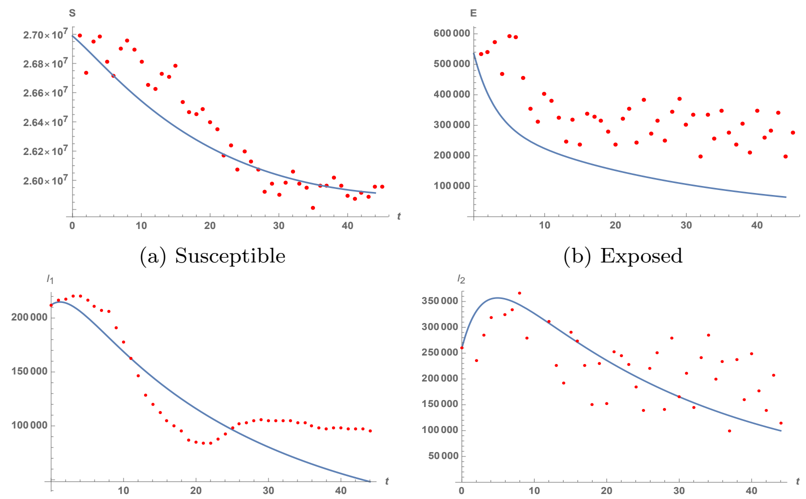
Some parameters in the system (1a)-(1e) are collected from literature. The birth and death rates were considered as the functions of the total population N between the years 2006 and 2019 in [8]. Since we work in relatively short time interval (45 days), these rates are assumed to be constants in this paper. Thus, the birth and death rates are fixed to $b = 0.000041, d = 0.0000157$, respectively. In this study, the recovery rate for diagnosed infected individuals $r_1$ and the recovery rate for undiagnosed infected individuals $r_2$ are assumed to be $r_1 = r_2 = 1/7$. Indeed, a recent work on the incubation period of COVID-19 reported that the mean incubation period of COVID-19 is 6.57 days ([27]). According to COVID-19 data of Türkiye, COVID-19 related death rate is $\theta = 0.489011 \times 10^3$ ([26]).

For three different scenarios, estimated parameters are shown in Table 4. Also, the numerical solutions for $S, E, I_1, I_2$ of system (1a)-(1e) with the best-fit parameters are plotted with the actual data in Figures 3-Fig.5 for each scenario. Estimated curve and observed data for $R$ is plotted separately in Fig. 6. We note that, the unknown parameters of the system (1a)-(1e) are estimated using the software Mathematica.

| | Scenario 1 | Scenario 2 | Scenario 3 |
|---|---|---|---|
| $\beta_1$ | 0.139302 | 0.061828 | 0.034483 |
| $\beta_2$ | 0.828683 | 0.481024 | 0.0001 |
| $\xi$ | 0.160594 | 0.25191 | 0.99 |
| $\gamma_1$ | 0.060718 | 0.0254791 | 0.004332 |
| $\gamma_2$ | 0.176248 | 0.134119 | 0.04908 |
| $\gamma_3$ | 0.011203 | 0.0001 | 0.0001 |
| $r_{-1}$ | 0.003392 | 0.00598 | 0.018366 |

Table 4: Estimated parameter values for Eqn. (1a)-(1e)

Due to the definition of the number of undiagnosed individuals, $I_2$, an increasing arises apparently in the total number of infected (diagnosed and undiagnosed) individuals in the transition from Scenario 1 to Scenario 3. While in the first day of the observation $I_2$ is obtained as 259516 for Scenario 1, it




(a) Susceptible (b) Exposed

(c) Diagnosed (d) Undiagnosed

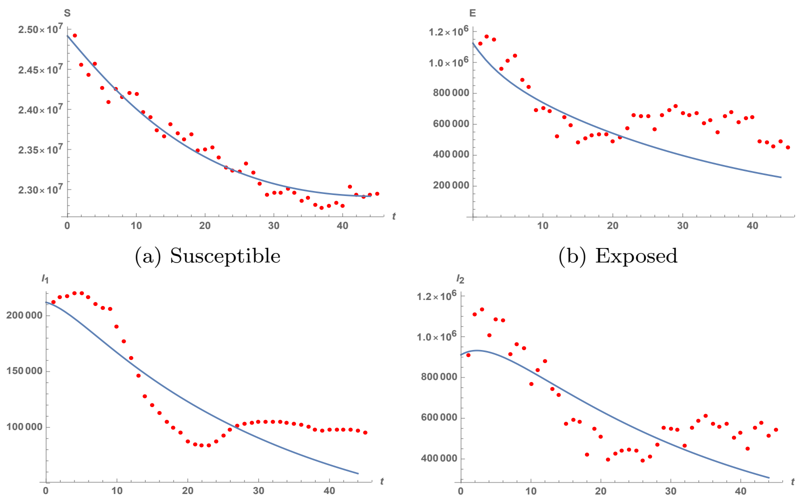
Figure 3: Numerical results of the fitted and observed values for Scenario 1. Red points are observed data and blue curves are the estimated curves of solution to the Model (1a)-(1e).

is obtained as 1655727 that is nearly 8 times of $I_1$ for Scenario 3 (See Fig. 2). On the other hand, when the estimated $\xi$ parameter values are compared from Scenario 1 through Scenario 3, there is an evident increase (See Table 4). Recall that, $\sqrt{\xi}$, $0 < \xi < 1$ represents the rate of protected individuals by social isolation in our model. Moreover, when social isolation decreases $\xi$ tends to 1. Indeed, these results are compatible with each other because, when people continue their social lives with less protection, the number of infected individuals in the society increases.

In this study, the coefficients of determination, $\mathscr{R}^2$ values are calculated for each scenario. The $\mathscr{R}^2$ value provides a measure for the goodness of the model to predict the data ([19]) and formulated as;

$$\mathscr{R}^2 = 1 - \frac{SS_{Res}}{SS_{Tot}},$$

where $SS_{Res}$ is the residual sum of squares and $SS_{Tot}$ is the total sum of squares. $\mathscr{R}^2$ values that are separately computed for each component of the Model (1a)-(1e) and their mean values are given in Table 5. The best mean $\mathscr{R}^2$ value 0.72 corresponds to Scenario 2 according to Table 5. $\mathscr{R}^2 = 0.72$ means that the model explains 72% of the data in Scenario 2. The mean $\mathscr{R}^2$




(a) Susceptible (b) Exposed

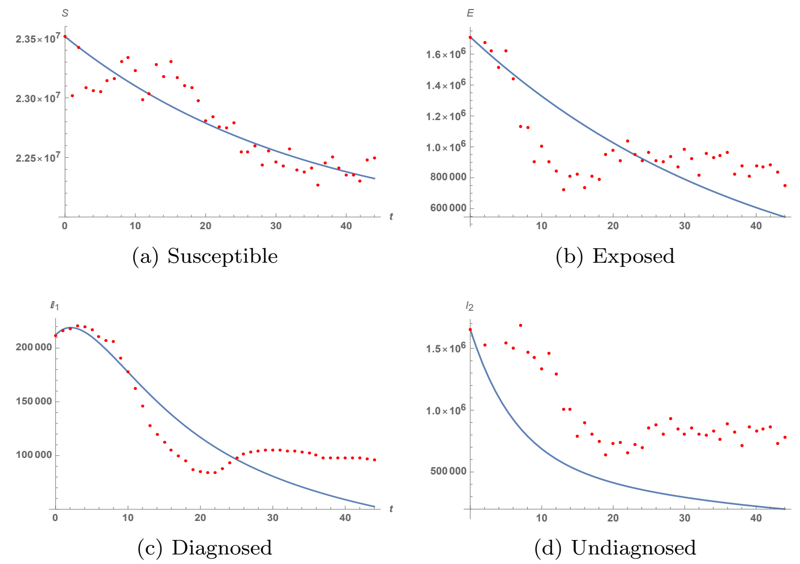
(c) Diagnosed (d) Undiagnosed

Figure 4: Numerical results of the fitted and observed values for Scenario 2. Red points are observed data and blue curves are the estimated curves of solution to the Model (1a)-(1e)

values in Table 5 show that the data of Scenario 2 is more accurate for the model (1a)-(1e) than the others. We note that if $\mathscr{R}^2 > 0.7$ then this value is generally considered as a strong effect size ([20]).

## 5 Conclusion

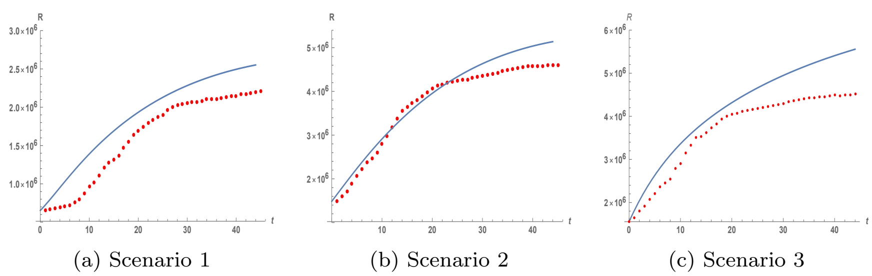
In this study we have considered a compartment model for COVID-19 with real data of Türkiye. Unknown parameters of the model have been estimated for three different scenarios. In our model infectious class has been divided into two sub-classes as diagnosed and undiagnosed. Data for diagnosed cases have been taken from real data of Türkiye as the total active cases. In this point the question ”How many undiagnosed individuals are present in the public?” arises. Many scientists agree that during respiratory viral outbreaks and pandemics, identified cases often represent only a small subset of what is really going on in a community ([12], [14]). The answer to this question changes clearly according to the socio-cultural, demographic and economic situations of the regions. In this context we generated three different data sets as Scenario 1, 2 and 3. Data fitting results for data of Türkiye showed that the best scenario is Scenario 2. According to this result, we conclude




(a) Susceptible (b) Exposed

(c) Diagnosed (d) Undiagnosed

Figure 5: Numerical results of the fitted and observed values for Scenario 3. Red points are observed data and blue curves are the estimated curves of solution to the Model (1a)-(1e)




(a) Scenario 1 (b) Scenario 2 (c) Scenario 3

Figure 6: The estimated curve of Recovered class of Eqn. (1b)-(1e) for three scenarios. Red points are real data and blue curves are estimated curves for populations of the Model (1b)-(1e)

that the undiagnosed case number is 4 to 6 times of the number of diagnosed cases which is compatible with a recent study [12]. In addition, both data and parameter results of Scenario 3 show that it is the worst scenario for Türkiye in terms of the huge number of infectious individuals. $\xi = 0.99$ is almost 1 in this scenario which means that there is almost no social isolation in the public.

| | $\mathscr{R}^2_{Scen1}$ | $\mathscr{R}^2_{Scen2}$ | $\mathscr{R}^2_{Scen3}$ |
|---|---|---|---|
| $S$ | 0.81 | 0.96 | 0.78 |
| $E$ | 0.19 | 0.48 | 0.51 |
| $I_1$ | 0.75 | 0.72 | 0.78 |
| $I_2$ | 0.32 | 0.5 | 0.27 |
| $R$ | 0.82 | 0.94 | 0.77 |
| mean $\mathscr{R}^2$ | 0.58 | 0.72 | 0.62 |

Table 5: $\mathscr{R}^2$ values of the numerical solutions corresponding to the estimated parameter values of the Model (1a)-(1e) for three scenario.

This result can be considered as another evidence for the importance of the regulations such as social distancing and wearing masks to avoid the damage of COVID-19 to the public health.

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The authors declare that they access and use the data reported to WHO by the Ministry of Health of Türkiye that can be found in the link below, that belongs to WHO, due to open access policy (CC BY-NC-SA 3.0 IGO) for noncommercial use: Microsoft Power BI
https://app.powerbi.com/view?r=eyJrIjoiYWRiZWVkNWUtNmM0Ni00MDA
wLTljYWMtN2EwNTM3YjQzYmRmIiwidCI6ImY2MTBjMGI3LWJkMjQtNG
IzOS04MTBiLTNkYzI4MGFmYjU5MCIsImMiOjh9

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