

A note on "The nearest symmetric fuzzy solution for a symmetric fuzzy linear system"

Ghassan Malkawi, Nazihah Ahmad and Haslinda Ibrahim

Abstract

This paper provides accurate approximate solutions for the symmetric fuzzy linear systems in (*Allahviranloo et al.*[1]).

1 Introduction

The following section reviews basic definitions of fuzzy theory, which will be needed in the sequel:

Definition 1.1. Let X be a universal set. Then, we define the fuzzy subset \tilde{A} of X by its membership function $\mu_{\tilde{A}} : X \to [0,1]$ which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1]; where the value $\mu_{\tilde{A}}(x)$ represents the grade of membership of x in \tilde{A} . A fuzzy set \tilde{A} is written as:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X, \ \mu_{\tilde{A}}(x) \in [0, 1]\}.$$

Definition 1.2. A fuzzy set \tilde{A} in $X = \mathbb{R}^n$ is convex fuzzy set if:

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$$\begin{aligned} \forall x_1, x_2 \in X, \forall \lambda \in [0,1], \\ \mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right). \end{aligned}$$

Definition 1.3. Let \tilde{A} be a fuzzy set defined on the set of real numbers \mathbb{R} . \tilde{A} is called normal fuzzy set if there exist $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$.

Definition 1.4. A fuzzy number is a normal and convex fuzzy set, with its membership function $\mu_{\tilde{A}}(x)$ defined in real line \mathbb{R} and piecewise continuous.

Definition 1.5. A fuzzy number $\tilde{A} = (a_1, a_2; \alpha, \beta)_{LR}$ is said to be an L-R fuzzy number, where its membership function satisfy

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{a_1-x}{\alpha}\right), & x \le a_1 \quad \alpha > 0, \\ 1, & a_1 \le x \le a_2, \\ R\left(\frac{x-a_2}{\beta}\right), & a_2 \le x \quad \beta > 0. \end{cases}$$

Where $a_1 \leq a_2$, and α and β are the left and right spreads, respectively; and the functions L(.), R(.), which are called left and right shape function,satisfying:

(1) L(.), R(.) are non-increasing from \mathbb{R}^+ to [0, 1],

(2) L(0) = R(0) = 1, L(1) = R(1) = 0.

Also, if $\alpha = \beta$ and L(x) = R(x) for all $x \in \mathbb{R}$ we say \overline{A} is a symmetric L-L fuzzy number.

Definition 1.6. (Allahviranloo et al.[1]) Let the shape functions L(.), R(.) are fixed. Consider two L-R fuzzy numbers as $\tilde{A} = (a_1, a_2; \alpha, \beta)$, and $\tilde{B} = (b_1, b_2; \gamma, \eta)$. We define the distance between \tilde{A} and \tilde{B} as follows:

$$d\left(\tilde{A},\tilde{B}\right) = \sqrt{\frac{[(a_1-b_1)-(\alpha-\gamma)]^2 + [(a_2-b_2)+(\beta-\eta)]^2 + (a_1-b_1)^2 + (a_2-b_2)^2}{4}}.$$

Definition 1.7. A vector $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$, where $\tilde{x}_i, 1 \leq i \leq n$ are L-R fuzzy numbers, is called an L-R fuzzy vector.

Definition 1.8. (Allahviranloo et al.[1]) For two L-R fuzzy vectors $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n), \tilde{Y} = (\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_n)$ we defined

$$D_p\left(\tilde{X},\tilde{Y}\right) = \left(\sum_{i=1}^n d^p\left(\tilde{x}_i,\tilde{y}_i\right)\right)^{\frac{1}{p}}.$$

as distance between them, where $p \geq 1$.

2 Numerical examples

In this section we provide proposed solutions for the examples in [1].

Example 2.1. (Allahviranloo et al.[1]) According to [1], the symmetric exact solution for S-L-FLS is:

$$\tilde{X}_{v} = \begin{bmatrix} \left(x_{1}^{1}, x_{2}^{1}; \alpha_{x}^{1}, \alpha_{x}^{1}\right) \\ \left(x_{1}^{2}, x_{2}^{2}; \alpha_{x}^{2}, \alpha_{x}^{2}\right) \\ \left(x_{1}^{3}, x_{2}^{3}; \alpha_{x}^{3}, \alpha_{x}^{3}\right) \end{bmatrix} = \begin{bmatrix} (1, 2; 2, 2) \\ (-1, 1; 1, 1) \\ (2, 4; 3, 3) \end{bmatrix}.$$

But \tilde{X}_v does not correspond to the system, for instance if $b_1^1 = 2$ is examined in vector \tilde{B} , we get :

$$(-1)(2) + (-1)(1) + (1)(2) = -1,$$

By using Definition 1.8. we produce $D_2\left(A\tilde{X}_v, \tilde{B}\right) = D_1\left(A\tilde{X}_v, \tilde{B}\right) = 3$. However, the symmetric exact solution corresponds to the system by solving the associated linear system is as follows:

$$\tilde{X}_{e} = \begin{bmatrix} \left(-\frac{5}{4}, -\frac{1}{4}; 2, 2\right) \\ \left(-7, -5; 1, 1\right) \\ \left(-\frac{13}{4}, -\frac{5}{4}; 3, 3\right) \end{bmatrix},$$
$$D_{p}\left(A\tilde{X}_{e}, \tilde{B}\right) = 0, \forall p \ge 1.$$

Example 2.2 (Allahviranloo et al.[1])

According to[1], the nearest symmetric approximate solution is:

$$\tilde{X}_{v} = \begin{bmatrix} \left(x_{1}^{1}, x_{2}^{1}; \alpha_{x}^{1}, \alpha_{x}^{1}\right) \\ \left(x_{1}^{2}, x_{2}^{2}; \alpha_{x}^{2}, \alpha_{x}^{2}\right) \\ \left(x_{1}^{3}, x_{2}^{3}; \alpha_{x}^{3}, \alpha_{x}^{3}\right) \end{bmatrix} = \begin{bmatrix} (2, 2, ; 2, 2) \\ (0.8333, 3.1667; 1, 1) \\ (0.5, 0.5; 1, 1) \end{bmatrix},$$

$$\text{then } A\tilde{X}_v = \begin{bmatrix} \left(b_1^1, b_2^1; \alpha_b^1, \alpha_b^1\right) \\ \left(b_1^2, b_2^2; \alpha_b^2, \alpha_b^2\right) \\ \left(b_1^3, b_2^3; \alpha_b^3, \alpha_b^3\right) \end{bmatrix} = \begin{bmatrix} (-3.8334, 0.8334; 5, 5) \\ (-4.6667, -2.3333; 4, 4) \\ (-2.6667, -0.3333; 6, 6) \end{bmatrix},$$
$$D_1(A\tilde{X}_v, B) = 1.1666, \\ D_2(A\tilde{X}_v, B) = 0.763763.$$

In fact, there are many nearer (symmetric or non-symmetric) approximate solutions based on the distance metric function in Definition 1.8.

In this note, we illustrate two cases for approximate fuzzy solutions.

Case 1: Symmetric approximate solution

The following L-L fuzzy vector \tilde{X}_1 is a symmetric approximate solution for the system, with distance metric function smaller than distance of solution \tilde{X}_v in [1].

Given

$$\tilde{X}_{1} = \begin{bmatrix} (2,2;1,1) \\ (0.75,3.25;0.75,0.75) \\ (0.5,0.5;2.5,2.5) \end{bmatrix}, \text{ then } A\tilde{X}_{1} = \begin{bmatrix} (-4,1;5,5) \\ (-4.75,-2.25;4.25,4.25) \\ (-2.75,-0.25;5.25,5.25) \end{bmatrix},$$

and the following result is obtained using Definition 1.8.

$$D_1(A\tilde{X}_1, \tilde{B}) = 0.707107,$$

 $D_2(A\tilde{X}_1, \tilde{B}) = 0.559017.$

Case2: Non-symmetric approximate solution

The following L-R fuzzy number vector \tilde{X}_2 is a non-symmetric approximate solution for the system, with distance metric function smaller than distance of solution \tilde{X}_v in [1].

Given

$$\tilde{X}_{2} = \begin{bmatrix} \left(\frac{13}{6}, \frac{13}{6}; \frac{7}{6}, \frac{5}{6}\right) \\ \left(\frac{5}{6}, \frac{10}{3}; \frac{5}{6}, \frac{2}{3}\right) \\ \left(\frac{1}{2}, \frac{1}{2}; \frac{5}{2}, \frac{5}{2}\right) \end{bmatrix}, \text{ then } A\tilde{X}_{2} = \begin{bmatrix} (-4, 1; 5, 5) \\ (-5, -\frac{5}{2}; 4, \frac{9}{2}) \\ \left(-3, -\frac{1}{2}; 5, \frac{11}{2}\right) \end{bmatrix},$$

and we produce the following results

$$D_1(A\tilde{X}_2, \tilde{B}) = 0.809017, D_2(A\tilde{X}_2, \tilde{B}) = 0.612372.$$

Note:

Our new solutions are obtained by using distance metric function which not only provides L-L fuzzy number vector, but also L-R fuzzy number vector.

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Ghassan Malkawi, College of Arts & Sciences, University Utara Malaysia, 06010 Sintok, Kedah, Malaysia. Email: gassan_malkawi@yahoo.com

Nazihah Ahmad, College of Arts & Sciences, University Utara Malaysia, 06010 Sintok, Kedah, Malaysia. Email: nazihah@uum.edu.my

Haslinda Ibrahim, College of Arts & Sciences, University Utara Malaysia, 06010 Sintok, Kedah, Malaysia. Email: linda@uum.edu.my

178