



HX -groups and Hypergroups

Piergiulio Corsini

Abstract

One considers the hypergroups associated with the HX -groups $\mathbf{Z}/n\mathbf{Z}$ and with the set of square matrices of order 2, with coefficients in $\mathbf{Z}/2\mathbf{Z}$ and one calculates their fuzzy grade.

1 Introduction

One finds in the literature on HX - groups some examples, but, at least those I have seen, they are all on infinite groups. So, it has been interesting to find some on finite groups. One has determined those corresponding with $\mathbf{Z}/n\mathbf{Z}$ and one has found the associated hypergroups (after the correspondence that, in [3] it is shown to exist).

Then one has considered the Fuzzy Grade [3, 4, 6, 7, 8, 9, 12, 13, 15] of these hypergroups, finding that for every n , it is always equal to one.

Finally, one has constructed the tables of associated hypergroups in the cases: $n = 8$, $n = 9$, $n = 12$, $n = 15$, $n = 16$ and of a set of subsets of $\mathbf{Z}_2^{(2,2)}$.

2 HX - groups on finite groups

Set $n \in \mathbf{N}$. We shall see that there is in $\mathbf{Z}/n\mathbf{Z}$ an HX -group, for every divisor of n .

Set $n = aq$ and let us consider the following subsets:

$$A_0 = [0, a, 2a, \dots, (q-1)a]$$

Key Words: HX -group, Fuzzy grade.
2010 Mathematics Subject Classification: Primary 20N20; Secondary 20N25.
Received: 11.05.2015
Accepted: 30.06.2015

$$\begin{aligned}
 A_1 &= [1, a + 1, 2a + 1, \dots, (q - 1)a + 1] \\
 A_2 &= [2, a + 2, 2a + 2, \dots, (q - 1)a + 2] \\
 &\dots \\
 A_s &= [s, a + s, 2a + s, \dots, (q - 1)a + s]
 \end{aligned}$$

where $s = a - 1$.

Let us denote \mathcal{G}_a the set

$$\mathcal{G}_a \subseteq \mathcal{P}^*(\mathbf{Z}/n\mathbf{Z}), \quad \mathcal{G}_a = \{A_i \mid 0 \leq i \leq a - 1\}$$

For all (i, j) ,

if $i + j < q$, set $A_i \circ A_j = A_{i+j}$;

if $i + j \geq q$, set $A_i \circ A_j = A_{(i+j)-q}$.

So, the structure (\mathcal{G}_a, \circ) is an abelian group and it is an *HX*-group in $\mathbf{Z}/n\mathbf{Z}$. We have the following structure described in the table here below:

\circ	A_0	A_1	A_2	\dots	A_{a-1}
A_0	A_0	A_1	A_2		A_{a-1}
A_1	A_1	A_2	A_3		A_0
A_2	A_2	A_3			A_1
\dots					
A_{a-1}	A_{a-1}				A_{a-2}

So, setting $K = \{A_i \mid 0 \leq i \leq a - 1\}$, we have that $(K, \circ) \simeq \mathbf{Z}/a\mathbf{Z}$.

Let q be a divisor of n , that is $n = qa$.

Then there is an *HX*-group \mathcal{G}_a^q in $\mathcal{P}^*(\mathbf{Z}/n\mathbf{Z})$ of q elements A_0, A_1, \dots, A_{q-1} , such that for all j , $|A_j| = a$, whence a Chinese hypergroupoid ${}_nH^a$ is constructed where for all x, y elements of ${}_nH^a$, we have $|x \oplus y| = a$.

We shall see now that (H, \oplus) is reproductible.

Let us consider the hypergroup associated with \mathcal{G}_a .

We know that $\forall (x, y) \in H^2 = (\mathbf{Z}/n\mathbf{Z})^2$, we have

$$x \oplus y = \bigcup_{x \in A, y \in B} A \cdot B.$$

We have $\bigcup_{i=0}^{a-1} A_i = H$. Indeed, $(q - 1)a + a - 1 = qa - 1$.

For t , such that $0 \leq t \leq qa - 1 = n - 1$, there exists (u, v) , such that $u \leq q - 1$, $v \leq a - 1$, $t = ua + v$.

Let us remark that the following implications hold:

$$(i_1) \quad t = 0 \Rightarrow t \in A_0$$

$$t = a \Rightarrow t \in A_0$$

$$(i'_1) \quad t = a + 1 \Rightarrow t \in A_1$$

$$(i_2) \quad t = a + 2 \Rightarrow t \in A_2$$

.....

$$(i'_2) \quad t = a + a - 1 \Rightarrow t \in A_{a-1}$$

$$(i_3) \quad t = 2a \Rightarrow t \in A_0$$

$$(i'_3) \quad t = 2a + 1 \Rightarrow t \in A_1$$

.....

$$(i''_3) \quad t = 2a + a - 1 \Rightarrow t \in A_{a-1}.$$

One can continue in the same way. So, for all k , we have

$$t = ka \Rightarrow t \in A_0$$

$$t = ka + 1 \Rightarrow t \in A_1$$

.....

$$t = ka + a - 1 \Rightarrow t \in A_{a-1}$$

and finally

$$t = (q - 1)a \Rightarrow t \in A_0$$

$$t = (q - 1)a + 1 \Rightarrow t \in A_1$$

$$t = (q - 1)a + 2 \Rightarrow t \in A_2$$

.....

$$t = (q - 1)a + a - 1 = qa - 1 \Rightarrow t \in A_{a-1}$$

$$t = qa = n \Rightarrow t \in A_0$$

So, $\bigcup_{j=0}^{a-1} A_j = H$.

We shall prove now that (H, \oplus) is a hypergroup, and then we shall see the table of (H, \oplus) .

We find that the hypergroupoid Q constructed on $\mathcal{G} = \{A_i\}_{i < q}$, $Q = (\mathcal{G}, \odot)$ is a group isomorphic with $\mathbf{Z}/q\mathbf{Z}$.

Therefore, $(\mathbf{Z}/n\mathbf{Z} : \mathcal{G}, \odot)$ is a HX -group. Let us show that (H, \oplus) is a hypergroup.

For all $(w, z) \in H^2$, $\exists x : z \in w \oplus x$. Indeed, $\exists A_k \in \mathcal{G}_a : w \in A_k, \exists h : z \in A_h$. Since \mathcal{G}_a is a group, there exists $A_r : A_k \odot A_r = A_h$, therefore if $x \in A_r$ we have $z \in A_h = A_k \odot A_r = w \oplus x$. So H is a hypergroup.

Let us consider for $\mathbf{Z}/n\mathbf{Z}$, $n = qa$. The set \mathcal{G}_a^n of subsets is an HX -group, such that $\forall A \in \mathcal{G}_a^n$ we have $|A| = a$. Then for all $x \in H$, we have $\mu_1(x) = 1/a$ so H_1 is total, whence $\partial H = 1$.

H_0	0	1	2	...	$a-1$	a	$a+1$	$a+2$...				$q-1$
0	A_0	A_1	A_2	...	A_{a-1}	A_0	A_1	A_2	...	A_{a-1}	A_0	...	A_{a-1}
1		A_2	A_3	...	A_0	A_1	A_2	A_3					A_{a-2}
2			A_4	...	A_1	A_2							
...						...							
a						A_{2a}							
$a+1$							A_0						
$a+2$								A_1					
...													
													A_{a-3}
													A_{a-2}

For $n = aq$, we have clearly

$$\forall s, A_s = [s, a + s, 2a + s, \dots, (q - 1)a + s]$$

.....

$$A_{a-1} = [a - 1, 2a - 1, \dots, (q - 1)a + a - 1].$$

3 The associated hypergroup for $n = 8$

Let us consider $\mathbf{Z}/8\mathbf{Z}$. Set $\mathcal{G}_2^8 = \{(0, 4), (1, 5), (2, 6), (3, 7)\}$. We have

We have clearly $A(0) = A(2) = A(4) = A(6) = 30/4$, and $\forall i \in \{0, 2, 4, 6\}$, $\mu_1(i) = 0.25$. Analogously, one finds $\forall j \in \{1, 3, 5, 7\}$, $\mu_1(j) = 0.25$.

4 The associated hypergroup for $n = 9$

Let us consider $\mathbf{Z}/9\mathbf{Z}$. Set $\mathcal{G}_3^9 = \{(0, 3, 6), (1, 4, 7), (2, 5, 8)\}$. We have the following

K_3^9	$(0, 3, 6)$	$(1, 4, 7)$	$(2, 5, 8)$
$(0, 3, 6)$	$(0, 3, 6)$	$(1, 4, 7)$	$(2, 5, 8)$
$(1, 4, 7)$		$(2, 5, 8)$	$(0, 3, 6)$
$(2, 5, 8)$			$(1, 4, 7)$

which is an HX -group. One obtains the hypergroup H_3^9 .

H_3^9	0	1	2	3	4	5	6	7	8
0	0, 3, 6	1, 4, 7	2, 5, 8	0, 3, 6	1, 4, 7	2, 5, 8	0, 3, 6	1, 4, 7	2, 5, 8
1		2, 5, 8	0, 3, 6	1, 4, 7	2, 5, 8	0, 3, 6	1, 4, 7	2, 5, 8	0, 3, 6
2			1, 4, 7	2, 5, 8	0, 3, 6	1, 4, 7	2, 5, 8	0, 3, 6	1, 4, 7
3				0, 3, 6	1, 4, 7	2, 5, 8	0, 3, 6	1, 4, 7	2, 5, 8
4					2, 5, 8	0, 3, 6	1, 4, 7	2, 5, 8	0, 3, 6
5						1, 4, 7	2, 5, 8	0, 3, 6	1, 4, 7
6							0, 3, 6	1, 4, 7	2, 5, 8
7								2, 5, 8	0, 3, 6
8									1, 4, 7

We find $A(0) = A(3) = A(6) = 1/3 + 4/3 + 7/3 + 8/3 + 5/3 + 2/3 = 9$. whence $\forall j, \mu_1(j) = 0.333$, whence ${}_1H_3^9$ is total, whence $\partial(H_3^9) = 1$.

5 The associated hypergroup for $n = 12$

Let us consider $\mathbf{Z}/12\mathbf{Z}$.

Set $\mathcal{G}_3^{12} = \{(0, 4, 8), (1, 5, 9), (2, 6, 10), (3, 7, 11)\}$. We have the following

K_3^{12}	(0, 4, 8)	(1, 5, 9)	(2, 6, 10)	(3, 7, 11)
(0, 4, 8)	(0, 4, 8)	(1, 5, 9)	(2, 6, 10)	(3, 7, 11)
(1, 5, 9)	(1, 5, 9)	(2, 6, 10)	(3, 7, 11)	(0, 4, 8)
(2, 6, 10)	(2, 6, 10)	(3, 7, 11)	(0, 4, 8)	(1, 5, 9)
(3, 7, 11)	(3, 7, 11)	(0, 4, 8)	(1, 5, 9)	(2, 6, 10)

Denote $A_0 = (0, 4, 8)$, $A_1 = (1, 5, 9)$, $A_2 = (2, 6, 10)$, $A_3 = (3, 7, 11)$. K_3^{12} is an HX-group. We obtain the Chinese hypergroup H_3^{12} .

H_3^{12}	0	1	2	3	4	5	6	7	8	9	10	11
0	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3
1	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0
2	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1
3	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2
4	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3
5	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0
6	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1
7	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2
8	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3
9	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0
10	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1
11	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2	A_3	A_0	A_1	A_2

Set $\mathcal{G}_4^{12} = \{(0, 3, 6, 9), (1, 4, 7, 10), (2, 5, 8, 11)\}$. We find

K_4^{12}	(0, 3, 6, 9)	(1, 4, 7, 10)	(2, 5, 8, 11)
(0, 3, 5, 9)	(0, 3, 6, 9)	(1, 4, 7, 10)	(2, 5, 8, 11)
(1, 4, 7, 10)	(1, 4, 7, 10)	(2, 5, 8, 11)	(0, 3, 6, 9)
(2, 5, 8, 11)	(2, 5, 8, 11)	(0, 3, 6, 9)	(1, 4, 7, 10)

So, K_4^{12} is clearly an HX-group and we obtain the Chinese hypergroup H_4^{12} .

First, we set $K_0 = (0, 3, 6, 9)$, $K_1 = (1, 4, 7, 10)$, $K_2 = (2, 5, 8, 11)$.

H_4^{12}	0	3	6	9	1	4	7	10	2	5	8	11
0	K_0	K_0	K_0	K_0	K_1	K_1	K_1	K_1	K_2	K_2	K_2	K_2
3	K_0	K_0	K_0	K_0	K_1	K_1	K_1	K_1	K_2	K_2	K_2	K_2
6	K_0	K_0	K_0	K_0	K_1	K_1	K_1	K_1	K_2	K_2	K_2	K_2
9	K_0	K_0	K_0	K_0	K_1	K_1	K_1	K_1	K_2	K_2	K_2	K_2
1	K_1	K_1	K_1	K_1	K_2	K_2	K_2	K_2	K_0	K_0	K_0	K_0
4	K_1	K_1	K_1	K_1	K_2	K_2	K_2	K_2	K_0	K_0	K_0	K_0
7	K_1	K_1	K_1	K_1	K_2	K_2	K_2	K_2	K_0	K_0	K_0	K_0
10	K_1	K_1	K_1	K_1	K_2	K_2	K_2	K_2	K_0	K_0	K_0	K_0
2	K_2	K_2	K_2	K_2	K_0	K_0	K_0	K_0	K_1	K_1	K_1	K_1
5	K_2	K_2	K_2	K_2	K_0	K_0	K_0	K_0	K_1	K_1	K_1	K_1
8	K_2	K_2	K_2	K_2	K_0	K_0	K_0	K_0	K_1	K_1	K_1	K_1
11	K_2	K_2	K_2	K_2	K_0	K_0	K_0	K_0	K_1	K_1	K_1	K_1

For $\forall j_0 \in K_0$, $A(j_0) = 16/4 + 16/4 + 16/4 = 12$, $q(j_0) = 48$, $\mu(j_0) = 0.25$;
 $\forall j_1 \in K_1$, $\mu(j_1) = 0.25$;
 $\forall j_2 \in K_2$, $\mu(j_2) = 0.25$.
 So H_2 is total, whence $\partial H_4^{12} = 1$.

Set $\mathcal{G}_6^{12} = \{(0, 2, 4, 6, 8, 10), (1, 3, 5, 7, 9, 11)\}$.
 Set $A_0 = (0, 2, 4, 6, 8, 10)$, $A_1 = (1, 3, 5, 7, 9, 11)$.

K_6^{12}	A_0	A_1
A_0	A_0	A_1
A_1	A_1	A_0

In the case \mathcal{G}_6^{12} we have $\forall j, \mu(j) = 1/6 = 0.16666$.
 In the case \mathcal{G}_4^{12} we have $\forall j, \mu(j) = 1/4 = 0.25$.
 In the case \mathcal{G}_3^{12} we have $\forall j, \mu(j) = 1/3 = 0.333$.
 In the case \mathcal{G}_2^{12} we have $\forall j, \mu(j) = 1/2 = 0.5$.

6 The associated hypergroup for $n = 15$

Let us consider $\mathbf{Z}/15\mathbf{Z}$. Set $\mathcal{G}_5^{15} = \{(0, 3, 6, 9, 12), (1, 4, 7, 10, 13), (2, 5, 8, 11, 14)\}$.

Set $A_0 = (0, 3, 6, 9, 12), A_1 = (1, 4, 7, 10, 13), A_2 = (2, 5, 8, 11, 14)$.

K_5^{15}	A_0	A_1	A_2
A_0	A_0	A_1	A_2
A_1	A_1	A_2	A_0
A_2	A_2	A_0	A_1

H_1^{15}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2
1	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0
2	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1
3	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2
4	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0
5	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1
6	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2
7	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0
8	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1
9	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2
10	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0
11	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1
12	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2
13	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0
14	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1	A_2	A_0	A_1

So, we have $\forall j, \mu(j) = 1/5 = 0.2$.

Set $\mathcal{G}_3^{15} = \{A_0^3, A_1^3, A_2^3, A_3^3, A_4^3\}$, where

$A_0^3 = (0, 5, 10), A_1^3 = (1, 6, 11), A_2^3 = (2, 7, 12), A_3^3 = (3, 8, 13),$

$A_4^3 = (4, 9, 14)$.

Notice that $K_3^{15} = \{A_i^2 \mid 0 \leq i \leq 4\}$ is an HX-group. Indeed,

K_3^{15}	A_0^3	A_1^3	A_2^3	A_3^3	A_4^3
A_0^3	A_0^3	A_1^3	A_2^3	A_3^3	A_4^3
A_1^3	A_1^3	A_2^3	A_3^3	A_4^3	A_0^3
A_2^3	A_2^3	A_3^3	A_4^3	A_0^3	A_1^3
A_3^3	A_3^3	A_4^3	A_0^3	A_1^3	A_2^3
A_4^3	A_4^3	A_0^3	A_1^3	A_2^3	A_3^3

H_3^{15}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4
1	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0
2	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1
3	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2
4	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3
5	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4
6	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0
7	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1
8	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2
9	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3
10	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4
11	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0
12	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1
13	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2
14	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3	A_4	A_0	A_1	A_2	A_3

So, we have $\forall j, \mu_1(j) = 1/3 = 0.333$.

7 The associated hypergroup for $n = 16$

Let us consider $\mathbf{Z}/16\mathbf{Z}$. Denote $A_0^{16,4} = A_0 = (0, 4, 8, 12)$, $A_1^{16,4} = A_1 = (1, 5, 9, 13)$, $A_2^{16,4} = A_2 = (2, 6, 10, 14)$, $A_3^{16,4} = A_3 = (3, 7, 11, 15)$.

We obtain the HX -group K_4^{16} , whence Chinese hypergroup H_4^{16} .

K_4^{16}	A_0^{16}	A_1^{16}	A_2^{16}	A_3^{16}
A_0^{16}	A_0	A_1	A_2	A_3
A_1^{16}	A_1	A_2	A_3	A_0
A_2^{16}	A_2	A_3	A_0	A_1
A_3^{16}	A_3	A_0	A_1	A_2

H_4^{16}	0	4	8	12	1	5	9	13	2	6	10	14	3	7	11	15
0	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3
4	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3
8	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3
12	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3
1	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0
5	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0
9	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0
13	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0
2	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1
6	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1
10	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1
14	A_2	A_2	A_2	A_2	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1
3	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2
7	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2
11	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2
15	A_3	A_3	A_3	A_3	A_0	A_0	A_0	A_0	A_1	A_1	A_1	A_1	A_2	A_2	A_2	A_2

We have $\forall j, \mu_1(j) = 1/4 = 0.25$.

Set $A_0^{16,8} = (0, 2, 4, 6, 8, 10, 12, 14)$, $A_1^{16,8} = (1, 3, 5, 7, 9, 11, 13, 15)$.

We obtain the HX-group K_8^{16} , whence the Chinese hypergroup H_8^{16} .

K_8^{16}	$A_0^{16,8}$	$A_1^{16,8}$
$A_0^{16,8}$	$A_0^{16,8}$	$A_1^{16,8}$
$A_1^{16,8}$	$A_1^{16,8}$	$A_0^{16,8}$

H_2^{16}	0	8	1	9	2	10	3	11	4	12	5	13	6	14	7	15
0	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7
8	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7
1	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0
9	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0
2	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1
10	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1
3	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2
11	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2
4	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3
12	B_4	B_4	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3
5	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4
13	B_5	B_5	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4
6	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5
14	B_6	B_6	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5
7	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6
15	B_7	B_7	B_0	B_0	B_1	B_1	B_2	B_2	B_3	B_3	B_4	B_4	B_5	B_5	B_6	B_6

We have $\forall j, \mu_1(j) = 0.5$.

8 HX-groupoids

The absence in the class of HX-groups associated with $\mathbf{Z}/n\mathbf{Z}$, $\forall n \in \mathbf{N}$ of elements $H^{(n)}$, such that the fuzzy grade $\partial H^{(n)}$ is greater than 1, could let think to be true for every finite hypergroupoid. I hit upon the idea of considering the multiplicative group (with respect with the product row \times column) of all square matrices of $\mathbf{Z}_2^{(2,2)}$ of order 2 with coefficients in $\mathbf{Z}/2\mathbf{Z}$.

$\mathbf{Z}_2^{(2,2)}$	1 0	0 1	1 1	1 1	1 0	0 1
	0 1	1 0	1 0	0 1	1 1	1 1
1 0	1 0	0 1	1 1	1 1	1 0	0 1
0 1	0 1	1 0	1 0	0 1	1 1	1 1
0 1	0 1	1 0	1 0	0 1	1 1	1 1
1 0	1 0	0 1	1 1	1 1	1 0	0 1
1 1	1 1	1 1	0 1	1 0	0 1	1 0
1 0	1 0	0 1	1 1	1 1	1 0	0 1
1 1	1 1	1 1	0 1	1 0	0 1	1 0
0 1	0 1	1 0	1 0	0 1	1 1	1 1
1 0	1 0	0 1	1 1	1 1	1 0	0 1
1 1	1 1	1 1	0 1	1 0	0 1	1 0
0 1	0 1	1 0	1 0	0 1	1 1	1 1
1 1	1 1	1 1	0 1	1 0	0 1	1 0

Then one has considered the hypergroupoid H_0 , as follows. First, we denote by B_i where $0 \leq i \leq 8$.

$$\begin{aligned}
B_0 &= \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \right\}, B_1 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right\} \\
B_2 &= \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}, B_3 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\} \\
B_4 &= \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\}, B_5 = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\} \\
B_6 &= \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\}, B_7 = \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\} \\
B_8 &= \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\}.
\end{aligned}$$

We construct H_0 , where for every pair (i, j) , we shall denote the hyperproduct $B_i \circ_0 B_j = \{B_h, B_k\}$ as $i \circ_0 j$. Then we shall calculate the membership function μ_1 associated with the hypergroupoid $(H_0; \circ_0)$. One obtains $(H_1; \circ_1)$ where \circ_1 is defined by

$$x \circ_1 y = \{z \mid \min\{\mu(x), \mu(y)\} \leq \mu(z) \leq \max\{\mu(x), \mu(y)\}\},$$

see [2], continuing in the same way,

H_0	0	1	2	3	4	5	6	7	8
0	0	0,5	2,4	0,5	2,4	5	6,7	6,7	2,4
1	0,1, 4,6	1	1,2, 5,7	1,2, 5,7	4	1,2, 5,7	0,1, 4,6	7	0,1, 4,6
2	0,7	2,6	2	0,7	4,5	4,5	6	0,7	2,6
3	0,7	0,5	1,2, 5,7	3,8	2,4	4,5	0,1, 4,6	6,7	3,8
4	0,1, 4,6	4,7	4	0,1, 4,6	1,2, 5,7	1,2, 5,7	7	0,1, 4,6	4,7
5	6,7	0,5	5	6,7	2,4	2,4	0	6,7	0,5
6	6	2,6	4,5	2,6	4,5	2	0,7	0,7	4,5
7	7	4,7	1,2, 5,7	4,7	1,2, 5,7	4	0,1, 4,6	0,1, 4,6	1,2, 5,7
8	0,1, 4,6	2,6	2,4	3,8	4,5	1,2, 5,7	6,7	0,7	3,8

We have clearly

$$A(0) = 2 \cdot 1 + 12 \cdot 1/2 + 10 \cdot 1/4 = 42/4$$

$$A(1) = 1 \cdot 1 + 19 \cdot 1/4 = 23/4$$

$$A(3) = A(8) = 4 \cdot 1/2 = 2$$

$$A(7) = 3 \cdot 1 + 17 \cdot 1/2 + 9 \cdot 1/4 = 55/4$$

$$A(2) = 2 \cdot 1 + 1 \cdot 1/2 + 10 \cdot 1/4 = 42/4$$

$$A(4) = 3 \cdot 1 + 17 \cdot 1/2 + 9 \cdot 1/4 = 55/4$$

$$A(6) = 2 \cdot 1 + 12 \cdot 1/2 + 10 \cdot 1/4 = 42/4$$

$$A(5) = 2 \cdot 1 + 12 \cdot 1/2 + 10 \cdot 1/4 = 42/4.$$

By consequence we have

$$q_1(0) = 24, q_1(1) = 20, q_1(3) = q_1(8) = 4$$

$$q_1(7) = 29, q_1(2) = 23, q_1(4) = 29, q_1(6) = 24$$

$$q_1(5) = 24, \text{whence } \mu_1(0) = \mu_1(2) = \mu_1(5) = \mu_1(6) = 0.4375$$

$$\mu_1(4) = \mu_1(7) = 0.47138, \mu_1(3) = \mu_1(8) = 0.5$$

$$\mu_1(1) = 0.2875.$$

One finds

H_1	1	0	2	5	6	4	7	3	8
1	1	0, 1, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 4, 5, 6, 7	0, 1, 2, 4, 5, 6, 7	H	H
0		0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 4, 5, 6, 7	0, 2, 4, 5, 6, 7	0, 2, 3, 4, 5, 6, 7, 8	0, 2, 3, 4, 5, 6, 7, 8
2			0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 4, 5, 6, 7	0, 2, 4, 5, 6, 7	0, 2, 3, 4, 5, 6, 7, 8	0, 2, 3, 4, 5, 6, 7, 8
5				0, 2, 5, 6	0, 2, 5, 6	0, 2, 4, 5, 6, 7	0, 2, 4, 5, 6, 7	0, 2, 3, 4, 5, 6, 7, 8	0, 2, 3, 4, 5, 6, 7, 8
6					0, 2, 5, 6	0, 2, 4, 5, 6, 7	0, 2, 4, 5, 6, 7	0, 2, 3, 4, 5, 6, 7, 8	0, 2, 3, 4, 5, 6, 7, 8
4						4, 7	4, 7	3, 4, 7, 8	3, 4, 7, 8
7							4, 7	3, 4, 7, 8	3, 4, 7, 8
3								3, 8	3, 8
8									3, 8

whence $A_2(1) = 1 + 8/5 + 4/7 + 4/9 = 2278/630$,

$$q_2(1) = 17, \mu_2(1) = 0.2127.$$

$A_2(0) = A_2(2) = A_2(5) = A_2(6) = 16/4 + 8/5 + 4/7 + 4/9 + 16/6 + 16/8 = 7108/630$,

$$q_2(0) = 48, \mu_2(0) = \mu_2(2) = \mu_2(5) = \mu_2(6) = 0.2350.$$

$$A_2(4) = A_2(7) = 4/2 + 16/6 + 4/7 + 4/9 + 8/4 + 16/8 = 6100/360,$$

$$q_2(4) = q_2(7) = 52, \mu_2(4) = \mu_2(7) = 0.18620.$$

$$A_2(3) = A_2(8) = 4/2 + 8/4 + 16/8 + 4/9 = 464/72,$$

$$q_2(3) = 32, \mu_2(3) = \mu_2(8) = 0.20139.$$

H_2	0	2	5	6	1	3	8	4	7
0	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	H	H
2	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	H	H
5	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	H	H
6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	H	H
1	0, 1, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 5, 6	1	1, 3, 8	1, 3, 8	1, 3, 4, 7, 8	1, 3, 4, 7, 8
3	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	1, 3, 8	3, 8	3, 8	3, 4, 7, 8	3, 4, 7, 8
8	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	1, 3, 8	3, 8	3, 8	3, 4, 7, 8	3, 4, 7, 8
4	H	H	H	H	1, 3, 4, 7, 8	3, 4, 7, 8	3, 4, 7, 8	4, 7	4, 7
7	H	H	H	H	1, 3, 4, 7, 8	3, 4, 7, 8	3, 4, 7, 8	4, 7	4, 7

$A_3(1) = 1 + 4/3 + 4/5 + 8/5 + 16/7 + 16/9 = 5542/630,$
 $q_3(1) = 49, \mu_3(1) = 0.179527.$
 $A_3(0) = A_3(2) = A_3(5) = A_3(6) = 16/4 + 8/5 + 16/7 + 16/9 = 6088/630,$
 $q_3(0) = q_3(2) = q_3(5) = q_3(6) = 56,$
 $\mu_3(0) = \mu_3(2) = \mu_3(5) = \mu_3(6) = 0.17256.$
 $A_3(3) = A_3(8) = 4/2 + 8/4 + 4/5 + 16/7 + 16/9 = 5584/630,$
 $q_3(3) = q_3(8) = 48, \mu_3(3) = \mu_3(8) = 0.184656.$
 $A_3(4) = A_3(7) = 4/2 + 8/4 + 4/5 + 16/9 = 4144/630,$
 $q_3(4) = q_3(7) = 32, \mu_3(4) = \mu_3(7) = 0.20555.$
 $\mu_3(0) = \mu_3(2) = \mu_3(5) = \mu_3(6) = 0.17256 < \mu_3(1) = 0.179527 <$
 $\mu_3(3) = \mu_3(8) = 0.184656 < \mu_3(4) = \mu_3(7) = 0.20555,$
 whence one finds H_3

H_3	0	2	5	6	1	3	8	4	7
0	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	H	H
2		0, 2, 5, 6	0, 2, 5, 6	0, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	H	H
5			0, 2, 5, 6	0, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	H	H
6				0, 2, 5, 6	0, 1, 2, 5, 6	0, 1, 2, 3, 5, 6, 8	0, 1, 2, 3, 5, 6, 8	H	H
1					1	1, 3, 8	1, 3, 8	1, 3, 4, 7, 8	1, 3, 4, 7, 8
3						3, 8	3, 8	3, 4, 7, 8	3, 4, 7, 8
8							3, 8	3, 4, 7, 8	3, 4, 7, 8
4								4, 7	4, 7
7									4, 7

One sees that $(H_3; \circ_3)$ coincides with $(H_2; \circ_2)$.

Therefore, one can conclude that the fuzzy grade of H_0 is 2.

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Piergiulio CORSINI,
Department of Polytecnic Engineering,
University of Udine,
Via delle Scienze 206, 33100 Udine, Italy.
Email: piergiuliacorsini@gmail.com

