



## On neighborhoods of functions associated with conic domains

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### Abstract

Let  $k-ST[A, B]$ ,  $k \geq 0$ ,  $-1 \leq B < A \leq 1$  be the class of normalized analytic functions defined in the open unit disk satisfying

$$\Re \left( \frac{(B-1) \frac{zf'(z)}{f(z)} - (A-1)}{(B+1) \frac{zf'(z)}{f(z)} - (A+1)} \right) > k \left| \frac{(B-1) \frac{zf'(z)}{f(z)} - (A-1)}{(B+1) \frac{zf'(z)}{f(z)} - (A+1)} - 1 \right|.$$

and let  $k-UCV[A, B]$ ,  $k \geq 0$ ,  $-1 \leq B < A \leq 1$  be the corresponding class satisfying

$$\Re \left( \frac{(B-1) \frac{(zf'(z))'}{f'(z)} - (A-1)}{(B+1) \frac{(zf'(z))'}{f'(z)} - (A+1)} \right) > k \left| \frac{(B-1) \frac{(zf'(z))'}{f'(z)} - (A-1)}{(B+1) \frac{(zf'(z))'}{f'(z)} - (A+1)} - 1 \right|.$$

For an appropriate  $\delta > 0$ , the  $\delta$  neighborhood of a function  $f \in k-UCV[A, B]$  is shown to consist of functions in the class  $k-ST[A, B]$ .

## 1 Introduction

Let  $\mathcal{A}$  denote the family of functions  $f$  of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad (1)$$

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Key Words: Analytic function,  $k$ -starlike functions,  $k$ -uniformly convex functions.  
2010 Mathematics Subject Classification: 30C45.  
Received: 8 May, 2014.  
Accepted: 30 June, 2014.

which are analytic in the open unit disk  $\mathcal{U} = \{z : |z| < 1\}$ . The classes  $S^*$  and  $C$  are the well-known classes of starlike and convex univalent functions respectively; for details, see[4].

Noor and Malik introduced and studied the class  $k - UCV[A, B]$  and the corresponding class  $k - ST[A, B]$  in [10] as following:

A function  $f(z) \in \mathcal{A}$  is said to be in the class  $k - ST[A, B]$ ,  $k \geq 0$ ,  $-1 \leq B < A \leq 1$ , if and only if,

$$\Re \left( \frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} \right) > k \left| \frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} - 1 \right|. \quad (2)$$

A function  $f(z) \in \mathcal{A}$  is said to be in the class  $k - UCV[A, B]$ ,  $k \geq 0$ ,  $-1 \leq B < A \leq 1$ , if and only if,

$$\Re \left( \frac{(B-1)\frac{(zf'(z))'}{f'(z)} - (A-1)}{(B+1)\frac{(zf'(z))'}{f'(z)} - (A+1)} \right) > k \left| \frac{(B-1)\frac{(zf'(z))'}{f'(z)} - (A-1)}{(B+1)\frac{(zf'(z))'}{f'(z)} - (A+1)} - 1 \right|. \quad (3)$$

It can be easily seen that

$$f(z) \in k - UCV[A, B] \iff zf'(z) \in k - ST[A, B]. \quad (4)$$

Special cases.

i.  $k - ST[1, -1] = k - ST$ ,  $k - UCV[1, -1] = k - UCV$ , the well-known classes of  $k$ -starlike and  $k$ -uniformly convex functions respectively, introduced by Kanas and Wisniowska [6, 7].

ii.  $k - ST[1 - 2\alpha, -1] = SD(k, \alpha)$ ,  $k - UCV[1 - 2\alpha, -1] = KD(k, \alpha)$ , the classes introduced by Shams et al. in [17].

iii.  $0 - ST[A, B] = S^*[A, B]$ ,  $0 - UCV[A, B] = C[A, B]$ , the well-known classes of Janowski starlike and Janowski convex functions respectively, introduced by Janowski [5].

Geometrically, if a function  $f(z) \in k - ST[A, B]$  then  $\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} = w$  takes all values from the domain  $\Omega_k$ ,  $k \geq 0$ , as

$$\Omega_k = \{w : \Re w > k |w - 1|\} \quad (5)$$

or equivalently

$$\Omega_k = \left\{ u + iv : u > k\sqrt{(u-1)^2 + v^2} \right\}. \quad (6)$$

The domain  $\Omega_k$  represents the right half plane for  $k = 0$ , a hyperbola for  $0 < k < 1$ , a parabola for  $k = 1$  and an ellipse for  $k > 1$ , for more details see [10].

Given  $\delta \geq 0$ , Ruscheweyh [16] defined the  $\delta$ -neighborhood  $N_\delta(f)$  of a function  $f(z) \in \mathcal{A}$  by

$$N_\delta(f) = \left\{ g(z) : g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \text{ and } \sum_{n=2}^{\infty} n |a_n - b_n| \leq \delta \right\}. \quad (7)$$

Ruscheweyh [16] proved among other results that  $N_{1/4}(f) \subset S^*$  for  $f \in \mathcal{C}$ . Sheil-Small and Silvia [18] introduced more general notions of neighborhood of an analytic function. These included noncoefficient neighbourhoods as well. Problems related to the neighborhoods and some other properties of analytic functions were considered by many authors, for example, see [1, 2, 3, 8, 9, 11, 12, 13, 14].

In this paper, the neighborhood  $N_\delta(f)$  for functions  $f(z) \in k-UCV[A, B]$  is investigated. It is shown that all functions  $g \in N_\delta(f)$  are in the class  $k-ST[A, B]$  for certain  $\delta > 0$ .

## 2 Main Results

In order to obtain the main results, a characterization of the class  $k-ST[A, B]$  in terms of the functions in another class  $k-ST'[A, B]$  is needed. For  $z \in \mathcal{U}$ , a function  $H_t$  is said to be in the class  $k-ST'[A, B]$  if the function  $H_t$  is of the form

$$H_t(z) = \frac{1}{(B-A) \left( 1 - kt \pm \sqrt{t^2 - (kt-1)^2 i} \right)} \times \left[ \frac{z \left[ B - 1 - (B+1) \left( kt \pm \sqrt{t^2 - (kt-1)^2 i} \right) \right]}{(1-z)^2} - \frac{z \left[ A - 1 - (A+1) \left( kt \pm \sqrt{t^2 - (kt-1)^2 i} \right) \right]}{1-z} \right] \quad (8)$$

where

$$\begin{cases} t \geq \frac{1}{k+1}, & \text{if } 0 \leq k \leq 1, \\ \frac{1}{k+1} \leq t \leq \frac{1}{k-1}, & \text{if } k > 1. \end{cases} \quad (9)$$

Recall that for any two functions  $f(z)$  and  $g(z)$  given by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad g(z) = z + \sum_{n=2}^{\infty} b_n z^n$$

the Hadamard product (or convolution) of  $f$  and  $g$  is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z) \tag{10}$$

**Lemma 2.1.** *A function  $f$  is in the class  $k - ST[A, B]$  if and only if*

$$\frac{1}{z}(f * H_t)(z) \neq 0, \quad z \in \mathcal{U} \tag{11}$$

for all  $H_t \in k - ST'[A, B]$ .

*Proof.* Let  $f \in k - ST[A, B]$ . Then the image of  $\mathcal{U}$  under

$$w = u + iv = \frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)}$$

lies in the conic type regions  $\Omega_k = \left\{ u + iv : u > k\sqrt{(u-1)^2 + v^2} \right\}$  so that for  $z \in \mathcal{U}$

$$\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} \neq kt \pm \sqrt{t^2 - (kt-1)^2}i \tag{12}$$

where

$$\begin{cases} t \geq \frac{1}{k+1}, & \text{if } 0 \leq k \leq 1, \\ \frac{1}{k+1} \leq t \leq \frac{1}{k-1}, & \text{if } k > 1. \end{cases} \tag{13}$$

Thus  $f \in k - ST[A, B]$  if and only if

$$\begin{aligned} & \left[ \left( B-1 - (B+1)(kt \pm \sqrt{t^2 - (kt-1)^2}i) \right) z f'(z) \right. \\ & \left. - \left( A-1 - (A+1)(kt \pm \sqrt{t^2 - (kt-1)^2}i) \right) f(z) \right] \\ & / \left[ z(B-A) \left( 1 - kt \pm \sqrt{t^2 - (kt-1)^2}i \right) \right] \neq 0 \end{aligned} \tag{14}$$

or equivalently

$$\frac{1}{z}(f * H_t)(z) \neq 0, \quad z \in \mathcal{U}$$

for all  $H_t \in k - ST'[A, B]$ . □

**Lemma 2.2.** *If*

$$H_t(z) = z + \sum_{n=2}^{\infty} h_n(t)z^n \in k - ST'[A, B], \quad z \in \mathcal{U} \quad (15)$$

then

$$|h_n(t)| < \frac{2n(t+1)}{|B-A|t} \quad (16)$$

where  $t$  is given in (13).

*Proof.* Writing  $H_t(z) = z + \sum_{n=2}^{\infty} h_n(t)z^n$ , and comparing coefficients of  $z^n$  in (8) we obtain

$$\begin{aligned} h_n(t) &= [n(B-1) - (A-1) - (n(B+1) - (A+1))kt \\ &\quad \pm (n(B+1) - (A+1))\sqrt{t^2 - (kt-1)^2 i}] \\ &\quad / \left[ (B-A) \left( 1 - kt \pm \sqrt{t^2 - (kt-1)^2 i} \right) \right] \end{aligned} \quad (17)$$

Thus

$$\begin{aligned} |h_n(t)|^2 &= [|n(B-1) - (A-1) - (n(B+1) - (A+1))kt \\ &\quad \pm (n(B+1) - (A+1))\sqrt{t^2 - (kt-1)^2 i}|^2] \\ &\quad / \left| (B-A) \left( 1 - kt \pm \sqrt{t^2 - (kt-1)^2 i} \right) \right|^2 \\ &= \left[ \{n(B-1) - (A-1) - [n(B+1) - (A+1)]kt\}^2 \right. \\ &\quad \left. + (n(B+1) - (A+1))^2 (t^2 - (kt-1)^2) \right] \\ &\quad / \left[ (B-A)^2 \left( (1-kt)^2 + t^2 - (kt-1)^2 \right) \right] \\ &= \frac{4(n-1)[(nB-A)(kt-1) + (n-1)kt] + [nB-A+n-1]^2 t^2}{(B-A)^2 t^2}. \end{aligned}$$

If  $-1 \leq B < A \leq 1$ ,  $n \geq 2$  then  $nB - A < n - 1$ , and from (13),  $kt - 1 < t$ . So

$$|h_n(t)|^2 \leq \frac{4(n-1)^2(t+1)^2}{(B-A)^2 t^2} < \frac{4n^2(t+1)^2}{(B-A)^2 t^2} \quad (18)$$

and

$$|h_n(t)| < \frac{2n(t+1)}{|B-A|t}.$$

□

**Lemma 2.3.** For a function  $f \in \mathcal{A}$  and  $\varepsilon \in \mathbb{C}$ , define the function  $F_\varepsilon$  by

$$F_\varepsilon = \frac{f(z) + \varepsilon z}{1 + \varepsilon}. \tag{19}$$

If for every  $\varepsilon$ ,  $|\varepsilon| < \delta$  we have  $F_\varepsilon \in k - ST[A, B]$ , then for every  $H_t \in k - ST'[A, B]$

$$\left| \frac{1}{z}(f * H_t)(z) \right| \geq \delta, \quad z \in \mathcal{U}. \tag{20}$$

*Proof.* If  $F_\varepsilon \in k - ST[A, B]$  for every  $\varepsilon$ ,  $|\varepsilon| < \delta$ , then by Lemma 2.1 for all  $H_t \in k - ST'[A, B]$

$$\frac{1}{z}(F_\varepsilon * H_t)(z) \neq 0, \quad z \in \mathcal{U}$$

or equivalently

$$\frac{(f * H_t)(z) + \varepsilon z}{(1 + \varepsilon)z} \neq 0. \tag{21}$$

Since  $|\varepsilon| < \delta$ , it is easily follows that

$$\left| \frac{1}{z}(f * H_t)(z) \right| \geq \delta.$$

□

**Theorem 2.1.** Let  $f \in \mathcal{A}$ ,  $\varepsilon \in \mathbb{C}$  and  $\delta > 0$ . If for every  $\varepsilon$ ,  $|\varepsilon| < \delta$ , we have  $F_\varepsilon \in k - ST[A, B]$  then

$$N_{\delta'}(f) \subset k - ST[A, B] \tag{22}$$

for

$$\delta' = \frac{|B - A|t}{2(t + 1)}\delta \tag{23}$$

where  $-1 \leq B < A \leq 1$  and  $\begin{cases} t \geq \frac{1}{k+1}, & \text{if } 0 \leq k \leq 1, \\ \frac{1}{k+1} \leq t \leq \frac{1}{k-1}, & \text{if } k > 1. \end{cases}$

*Proof.* Let  $g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in N_{\delta'}(f)$ . For any  $H_t \in k - ST'[A, B]$ ,

$$\begin{aligned} \left| \frac{1}{z}(g * H_t)(z) \right| &= \left| \frac{1}{z}(f * H_t)(z) + \frac{1}{z}((g - f) * H_t)(z) \right| \\ &\geq \left| \frac{1}{z}(f * H_t)(z) \right| - \left| \frac{1}{z}((g - f) * H_t)(z) \right| \end{aligned}$$

From lemma 2.3,

$$\begin{aligned} \left| \frac{1}{z}(g * H_t)(z) \right| &\geq \delta - \left| \sum_{n=2}^{\infty} \frac{(b_n - a_n) h_n(t) z^n}{z} \right| \\ &\geq \delta - \sum_{n=2}^{\infty} |(b_n - a_n)| |h_n(t)| \\ &\geq \delta - \frac{2(t+1)}{|B-A|t} \sum_{n=2}^{\infty} n |(b_n - a_n)|. \end{aligned}$$

Using Lemma 2.2 and noting that  $g \in N_{\delta'}(f)$  and whence  $\sum_{n=2}^{\infty} n |(b_n - a_n)| < \delta'$ , thus

$$\left| \frac{1}{z}(g * H_t)(z) \right| \geq \delta - \frac{2(t+1)}{|B-A|t} \delta' = 0$$

Therefore  $|\frac{1}{z}(g * H_t)(z)| \neq 0$  in  $\mathcal{U}$  for all  $H_t \in k - ST'[A, B]$  if

$$\delta' = \frac{|B-A|t}{2(t+1)} \delta$$

By Lemma 2.1  $g \in k - ST[A, B]$ . This proves that  $N_{\delta'}(f) \subset k - ST[A, B]$ .  $\square$

**Lemma 2.4.** ([15]) *If  $\phi$  is a convex univalent function with  $\phi(0) = 0 = \phi'(0) - 1$  in  $\mathcal{U}$  and  $g$  is starlike univalent in  $\mathcal{U}$ , then for each analytic function  $F$  in  $\mathcal{U}$ ,*

$$\frac{(\phi * Fg)(z)}{(\phi * g)(z)} \subset \overline{co}F(\mathcal{U}), \quad (z \in \mathcal{U}) \tag{24}$$

where  $\overline{co}$  stands for the closed convex hull.

**Lemma 2.5.** *If  $\phi(z) \in C$ ,  $f(z) \in k - ST[A, B]$  and  $g(z) = (B + 1)zf'(z) - (A + 1)f(z) \in S^*$  then  $(\phi * f)(z) \in k - ST[A, B]$ .*

*Proof.* Assume  $F(z) = \frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)}$ . Then for  $G(z) = (\phi * f)(z)$ , we have  $zG'(z) = \phi(z) * zf'(z)$ .

Hence

$$\begin{aligned}
 \frac{(B-1)\frac{zG'(z)}{G(z)} - (A-1)}{(B+1)\frac{zG'(z)}{G(z)} - (A+1)} &= \frac{(B-1)\frac{\phi(z)*zf'(z)}{(\phi*f)(z)} - (A-1)}{(B+1)\frac{\phi(z)*zf'(z)}{(\phi*f)(z)} - (A+1)} \\
 &= \frac{\phi(z)*[(B-1)zf'(z) - (A-1)f(z)]}{\phi(z)*[(B+1)zf'(z) - (A+1)f(z)]} \\
 &= \frac{\phi(z)*\left[\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)}\right] [(B+1)zf'(z) - (A+1)f(z)]}{\phi(z)*[(B+1)zf'(z) - (A+1)f(z)]} \\
 &= \frac{\phi(z)*(Fg)(z)}{\phi(z)*g(z)}. \tag{25}
 \end{aligned}$$

By Lemma 2.4 the image of  $\mathcal{U}$  under  $\frac{\phi(z)*(Fg)(z)}{\phi(z)*g(z)}$  is a subset of the convex hull of  $F(\mathcal{U})$ . That means  $(\phi * f)(z) \in k - ST[A, B]$ .  $\square$

**Theorem 2.2.** *If  $f \in k-UCV[A, B]$  and  $g(z) = (B+1)zf'(z) - (A+1)f(z) \in C$  then  $F_\varepsilon = \frac{f(z)+\varepsilon z}{1+\varepsilon} \in k - ST[A, B]$  for  $|\varepsilon| < \frac{1}{4}$ .*

*Proof.* Let  $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in k - UCV[A, B]$ . Then

$$\begin{aligned}
 F_\varepsilon &= \frac{f(z) + \varepsilon z}{1 + \varepsilon} \\
 &= \frac{z(1 + \varepsilon) + \sum_{n=2}^{\infty} a_n z^n}{1 + \varepsilon} \\
 &= \frac{f(z) * \left\{ z(1 + \varepsilon) + \sum_{n=2}^{\infty} z^n \right\}}{1 + \varepsilon} \\
 &= f(z) * \frac{\left( z - \frac{\varepsilon}{1+\varepsilon} z^2 \right)}{1 - z} = f(z) * h(z)
 \end{aligned}$$

where  $h(z) = \frac{z - \frac{\varepsilon}{1+\varepsilon} z^2}{1 - z}$ . Now,

$$\frac{zh'(z)}{h(z)} = \frac{1 - 2\rho z + \rho z^2}{(1 - \rho z)(1 - z)}, \quad \text{where } \rho = \frac{\varepsilon}{1 + \varepsilon}.$$

Hence  $|\rho| = \frac{|\varepsilon|}{1+|\varepsilon|} < \frac{1}{3}$  gives  $|\varepsilon| < \frac{1}{4}$ . Thus

$$\Re \left( \frac{zh'(z)}{h(z)} \right) \geq \frac{1 - 2|\rho||z| - |\rho||z|^2}{(1 - |\rho||z|)(1 + |z|)} > 0. \tag{26}$$



This inequality holds for all  $|\rho| < \frac{1}{3}$  which is true for  $|\varepsilon| < \frac{1}{4}$ . Therefore  $h$  is starlike in  $\mathcal{U}$  and so

$$\int_0^z \frac{h(t)}{t} dt = z + \sum_{n=2}^{\infty} \frac{c_n}{n} z^n = h(z) * \log\left(\frac{1}{1-z}\right) \tag{27}$$

is convex for  $|\varepsilon| < \frac{1}{4}$

$$(f * h)(z) = z f'(z) * \left[ h(z) * \log\left(\frac{1}{1-z}\right) \right] \tag{28}$$

$f(z) \in k\text{-UCV}[A, B] \implies z f'(z) \in k\text{-ST}[A, B]$  and  $h(z) * \log\left(\frac{1}{1-z}\right) \in C$ .

Also  $g(z) \in C$  implies that  $z g'(z) \in S^*$ . By Lemma 2.5, we have

$$z f'(z) * \left[ h(z) * \log\left(\frac{1}{1-z}\right) \right] \in k\text{-ST}[A, B].$$

Thus

$$(f * h)(z) = \frac{f(z) + \varepsilon z}{1 + \varepsilon} \in k\text{-ST}[A, B].$$

□

**Theorem 2.3.** *Let  $f \in k\text{-UCV}[A, B]$  then*

$$N_{\delta'}(f) \subset k\text{-ST}[A, B] \tag{29}$$

for

$$\delta' = \frac{|B-A|t}{8(t+1)} \tag{30}$$

where  $-1 \leq B < A \leq 1$  and  $\begin{cases} t \geq \frac{1}{k+1}, & \text{if } 0 \leq k \leq 1, \\ \frac{1}{k+1} \leq t \leq \frac{1}{k-1}, & \text{if } k > 1. \end{cases}$

*Proof.* Let  $f \in k\text{-UCV}[A, B]$  then we get the result by taking  $\delta = \frac{1}{4}$  in Theorem 2.1 and using Theorem 2.2. □

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