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A new characterization of A_7 and A_8

Alireza Khalili Asboei, Seyed Sadegh Salehi Amiri, Ali Iranmanesh and Abolfazl Tehranian

Abstract

Let G be a finite group and $\pi_e(G)$ be the set of element orders of G. Let $k \in \pi_e(G)$ and m_k be the number of elements of order k in G. Set $\mathrm{nse}(G){:=}\{m_k | k \in \pi_e(G)\}$. It is proved that A_n are uniquely determined by $\mathrm{nse}(A_n)$, where $n \in \{4, 5, 6\}$. In this paper, we prove that if G is a group such that $\mathrm{nse}(G){=}\mathrm{nse}(A_n)$ where $n \in \{7, 8\}$, then $G \cong A_n$.

1 Introduction

If n is an integer, then we denote by $\pi(n)$ the set of all prime divisors of n. Let G be a finite group. Denote by $\pi(G)$ the set of primes p such that G contains an element of order p. Also the set of element orders of G is denoted by $\pi_e(G)$. A finite group G is called a simple K_n-group, if G is a simple group with $|\pi(G)| = n$.

Set $m_i{=}m_i(G){=}|\{g \in G|$ the order of g is $i\}|$. In fact, m_i is the number of elements of order i in G, and $\mathrm{nse}(G){:=}\{m_i | i \in \pi_e(G)\}$, the set of sizes of elements with the same order. For the set $\mathrm{nse}(G)$, the most important problem is related to Thompson's problem. In 1987, J. G. Thompson posed a very interesting problem related to algebraic number fields as follows. For each finite group G and each integer $d \geq 1$, let $G(d) = \{x \in G| \ x^d = 1\}$. Defining G_1 and G_2 is of the same order type if, and only if, $|G_1(d)| = |G_2(d)|$, $d = 1$,

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2, 3, $\cdots$. Suppose G_1 and G_2 are of the same order type. If G_1 is solvable, is G_2 necessarily solvable?([10, Problem 12.37])

We know that if groups G_1 and G_2 are of the same order type, then $|G_1| = |G_2|$ and $\mathrm{nse}(G_1) = \mathrm{nse}(G_2)$. So it is natural to investigate the Thompson's Problem by $|G|$ and $\mathrm{nse}(G)$.
In [4], [2], [3] and [1], it is proved that all simple K_4- groups, symmetric groups S_r where r is prime, sporadic simple groups and $L_2(p)$ where p is prime, can be uniquely determined by $\mathrm{nse}(G)$ and the order of G. In [9] and [8], it is proved that the groups A_4, A_5 and A_6, $L_2(q)$ for $q \in \{7, 8, 11, 13\}$ are uniquely determined by only $\mathrm{nse}(G)$. In [9], the authors gave the following problem:

Problem: Is a group G isomorphic to A_n $(n \geq 4)$ if and only if $\mathrm{nse}(G)$ $= \mathrm{nse}(A_n)$?

In this paper, we give a positive answer to this problem and show that the alternating group A_n is characterizable by only $\mathrm{nse}(G)$ for $n \in \{7, 8\}$. In fact the main theorem of our paper is as follows:

Main Theorem: Let G be a group such that $\mathrm{nse}(G)$=$\mathrm{nse}(A_n)$, where $n \in \{7, 8\}$. Then $G \cong A_n$.

We note that there are finite groups which are not characterizable even by $\mathrm{nse}(G)$ and $|G|$. In 1987, Thompson gave an example as follows: Let G_1= $(C_2 \times C_2 \times C_2 \times C_2) \rtimes A_7$ and $G_2 = L_3(4) \rtimes C_2$ be the maximal subgroups of M_{23}. Then $\mathrm{nse}(G_1) = \mathrm{nse}(G_2)$ and $|G_1|$=$|G_2|$, but $G_1 \ncong G_2$. Throughout this paper, we denote by ϕ the Euler totient function. If G is a finite group, then we denote by P_q a Sylow $q-$ subgroup of G and $n_q(G)$ is the number of Sylow $q-$subgroup of G, that is, $n_q(G)$=$|\mathrm{Syl}_q(G)|$. All other notations are standard and we refer to [7], for example.

2 Main Results

In this section, for the proof of main theorem, we need the following Lemmas:
Lemma 2.1. [5] Let G be a finite group and m be a positive integer dividing $|G|$. If $L_m(G) = \{g \in G | g^m = 1\}$, then $m \mid |L_m(G)|$.

Lemma 2.2. [6] Let G be a finite group and $p \in \pi(G)$ be odd. Suppose that P is a Sylow $p-$subgroup of G and $n = p^s m$, where $(p, m) = 1$. If P is not cyclic and $s > 1$, then the number of elements of order n is always a multiple of p^s.

Lemma 2.3. [9] Let G be a group containing more than two elements.

Let $k \in \pi_e(G)$ and m_k be the number of elements of order k in G. If $s = sup\{m_k | k \in \pi_e(G)\}$ is finite, then G is finite and $|G| \leq s(s^2-1)$.

Let G be a group such that nse(G)=nse(A_n), where $n \in \{7, 8\}$. By Lemma 2.3, we can assume that G is finite. Let m_n be the number of elements of order n. We note that $m_n = k\phi(n)$, where k is the number of cyclic subgroups of order n in G. Also we note that if $n > 2$, then $\phi(n)$ is even. If $n \mid |G|$, then by Lemma 2.1 and the above notation we have

$$\begin{cases} \phi(n) \mid m_n \\ \\ n \mid \sum_{d|n} m_d \end{cases} \qquad (*)$$

In the proof of the main theorem, we often apply $(*)$ and the above comments.

3 Proof of the Main Theorem

Let G be a group such that nse(G)=nse(A_7)=$\{1, 105, 210, 350, 504, 630, 720\}$. First we prove that $\pi(G) \subseteq \{2, 3, 5, 7\}$. Since $105 \in$nse(G), it follows that by $(*)$, $2 \in \pi(G)$ and $m_2 = 105$. Let $2 \neq p \in \pi(G)$. By $(*)$, $p \mid (1 + m_p)$ and $(p-1) \mid m_p$, which implies that $p \in \{3, 5, 7, 211, 631\}$. If $211 \in \pi(G)$, then by $(*)$, $m_{211} = 210$. On the other hand, by $(*)$ we conclude that if $422 \in \pi_e(G)$, then $m_{422} = 210$ or 630 and $422 \mid (1+m_2+m_{211}+m_{422})$, and hence $422 \mid 526$ or $422 \mid 946$, which is a contradiction. Thus $422 \notin \pi_e(G)$. Since $422 \notin \pi_e(G)$, the group P_{211} acts fixed point freely on the set of elements of order 2, and so $|P_{211}| \mid m_2$, which is a contradiction. Hence $211 \notin \pi_e(G)$. Similar to the above discussion $631 \notin \pi(G)$.

If 3, 5 and $7 \in \pi(G)$, then $m_3 = 350$, $m_5 = 504$ and $m_7 = 720$, by $(*)$. Also we can see easily that G does not contain any elements of order 35, 81, 64, 125 and 343. Similarly, we can see that if 10, 14, 15, $21 \in \pi_e(G)$, then $m_{10} = 720$, $m_{14} \in \{210, 504\}$, $m_{15} = 720$ and $m_{21} = 504$.

If $2^a \times 3^b \in \pi_e(G)$, then $2^a \times 3^{b-1} \mid m_{2^a\times 3^b}$. Hence $1 \leq a \leq 3$ and $1 \leq b \leq 3$.

If $2^c \times 5^d \in \pi_e(G)$, then $2^{c+1} \times 5^{d-1} \mid m_{2^c\times 5^d}$. Hence $1 \leq c \leq 3$ and $1 \leq d \leq 2$.

If $2^e \times 7^f \in \pi_e(G)$, then $2^e \times 3 \times 7^{f-1} \mid m_{2^e\times 7^f}$. Hence $1 \leq e \leq 3$ and $1 \leq f \leq 2$.

If $3^k \times 5^h \in \pi_e(G)$, then $2^3 \times 3^{k-1} \times 5^{h-1} \mid m_{3^k\times 5^h}$. Hence $1 \leq k \leq 3$ and $1 \leq h \leq 2$.

If $3^l \times 7^m \in \pi_e(G)$, then $2^2 \times 3^l \times 7^{m-1} \mid m_{3^l\times 7^m}$. Hence $1 \leq l \leq 2$ and $1 \leq m \leq 2$.

In follow, we show that $\pi(G)$ could not be the sets $\{2\}$, $\{2, 3\}$, $\{2, 3, 7\}$ and $\{2, 3, 5\}$, and $\pi(G)$ must be equal to $\{2, 3, 5, 7\}$.

Case a. If $\pi(G) = \{2\}$, then $\pi_e(G) \subseteq \{1, 2, 2^2, 2^3, 2^4, 2^5\}$. Since $\mathrm{nse}(G)$ has seven elements, this case impossible.

Case b. We know that $2 \in \pi(G)$. We claim that $3 \in \pi(G)$. Suppose that $3 \notin \pi(G)$. If $5, 7 \notin \pi(G)$, then by Case a, we get a contradiction. Hence 5 or $7 \in \pi(G)$.

Let $5 \in \pi(G)$. Since $125 \notin \pi_e(G)$, $\exp(P_5) = 5$ or 25. If $\exp(P_5) = 5$, then by Lemma 2.1, $|P_5| \mid (1 + m_5) = 505$. Hence $|P_5| = 5$. Then $n_5 = m_5/\phi(5) = 504/4 \mid |G|$, a contradiction. If $\exp(P_5) = 25$, then $|P_5| \mid (1+m_5+m_{25})$. Hence $|P_5| = 25$ and $n_5 = m_{25}/20 \mid |G|$. Since $m_{25} = 720$, we get a contradiction. Thus $5 \notin \pi(G)$.

Let $7 \in \pi(G)$. Since $7^3 \notin \pi_e(G)$, $\exp(P_7) = 7$ or 49. If $\exp(P_7) = 7$, then by Lemma 2.1, $|P_7| \mid (1 + m_7) = 721$. Hence $|P_7| = 7$ and $n_7 = m_7/\phi(7) \mid |G|$, which is a contradiction.

If $\exp(P_7) = 49$, then $|P_7| \mid (1 + m_7 + m_{49})$. Hence $|P_7| = 49$. Since $m_{49} \in \{210, 504\}$, $n_7 = m_{49}/\phi(49) = 5$ or 12. By Sylow's theorem $n_7 = 7k+1$ for some k, since $n_7 = 5$ or 12, we get a contradiction. Thus $3 \in \pi(G)$.

Case c. Let $\pi(G) = \{2, 3\}$. Since $3^4 \notin \pi_e(G)$, $\exp(P_3) = 3$, 3^2 or 3^3. If $\exp(P_3) = 3$, $|P_3| \mid (1 + m_3) = 351$, by Lemma 2.1. Thus $|P_3| \mid 3^3$. If $|P_3| = 3$, then $n_3 = m_3/2 \mid |G|$, a contradiction. If $|P_3| > 3$, then since $\exp(P_3) = 3$, $|\pi_e(G)| \leq 11$. Therefore $|G| = 2^m \times 3^n = 2520 + 350k_1 + 504k_2 + 720k_3 + 630k_4 + 210k_5$, where m, n, k_1, k_2, k_3, k_4 and k_5 are non-negative integers and $0 \leq k_1 + k_2 + k_3 + k_4 + k_5 \leq 4$. It is clear that $|G| \leq 5400$. If $n = 2$, then $m = 9$. It easy to check that the above equation has no solution. If $n = 3$, then $m = 7$, arquing as above, the equation has no solution. Therefore $\exp(P_3) \neq 3$.

If $\exp(P_3) = 9$, by Lemma 2.1, $|P_3| \mid (1+m_3+m_9)$. Since $m_9 \in \{504, 630, 720\}$, $|P_3| = 9$. Hence $n_3 = m_9/6 \mid |G|$, a contradiction.

If $\exp(P_3) = 27$, then since $m_{27} \in \{504, 630, 720\}$, $|P_3| \mid 3^5$. If $|P_3| = 27$, then $n_9 = m_{27}/18 \mid |G|$, a contradiction. If $|P_3| = 81$ or 243, then by Lemma 2.2, $3^3 \mid m_{27}$, a contradiction.

Case d. Let $\pi(G) = \{2, 3, 7\}$. Since $7^3 \notin \pi_e(G)$, $\exp(P_7) = 7$ or 7^2. If $\exp(P_7) = 7$, then $|P_7| \mid (1 + m_7) = 721$. Hence $|P_7| = 7$ and $n_7 = m_7/6 = 120 \mid |G|$, a contradiction.

If $\exp(P_7) = 49$, then $|P_7| \mid (1 + m_7 + m_{49})$. Thus $|P_7| = 49$ and $n_7 = m_{49}/42 = 5$ or 12. By Sylow's theorem, we get a contradiction.

Case e. Let $\pi(G) = \{2, 3, 5\}$. Since $125 \notin \pi_e(G)$, $\exp(G) = 5$ or 25. If $\exp(G) = 5$, then $|P_5| \mid (1 + m_5) = 505$. Hence $|P_5| = 5$ and $n_5 = m_5/4 = 126 \mid |G|$, which is a contradiction.

If $\exp(P_5) = 25$, then $|P_5| \mid (1 + m_5 + m_{25}) = 1225$. Hence $|P_5| = 25$ and then the group P_5 is cyclic. Thus $n_5 = m_{25}/20 = 36$. Since a cyclic group of order 25 has 4 elements of order 5, $m_5 \leq 4\times n_5 = 144$, which is a contradiction.

Case f. Let $\pi(G) = \{2, 3, 5, 7\}$. Since $35 \notin \pi_e(G)$, the group P_7 acts fixed point freely on the set of elements of order 5, and so $|P_7| \mid m_5 = 504$, which implies that $|P_7| = 7$. Similarly, $|P_5| = 5$.

We know that if P and Q are Sylow $7-$subgroups of G, then P and Q are conjugate, which implies that $C_G(P)$ and $C_G(Q)$ are conjugate in G. Therefore $m_{21} = \phi(21) \cdot n_7 \cdot k$, where k is the number of cyclic subgroups of order 3 in $C_G(P_7)$. Since $n_7 = m_7/\phi(7) = 120$, $2 \times 720 \mid m_{21}$, which is a contradiction. Hence $21 \notin \pi_e(G)$. Similarly, $10 \notin \pi_e(G)$.

Since $21 \notin \pi_e(G)$, the group P_3 acts fixed point freely on the set of elements of order 7, and $|P_3| \mid m_7$. Thus $|P_3| \mid 9$. Also since $10 \notin \pi_e(G)$, $|P_2| \mid m_5 = 504$, and so $|P_2| \mid 2^3$.

If $|P_3| = 3$, then $|G| = 2^m\times 105$ and $m \leq 3$. On the other hand, $2520 \leq |G|$, a contradiction. Therefore $|P_3| = 9$ and $|G| = 2^m \times 315$ where $m \leq 3$. Since $2520 \leq |G|$, $m = 3$ and then $|G| = 2520 = |A_7|$. By [4], since A_7 is a simple K_4-group, $G \cong A_7$.

Now suppose that G be a group such that nse(G)=nse(A_8)= $\{$ 1, 315, 1232, 1344, 2688, 3780, 5040, 5760 $\}$. First we prove that $\pi(G) \subseteq \{2, 3, 5, 7\}$. Since $315 \in$nse(G), it follows that by $(*)$, $2 \in \pi(G)$ and $m_2 = 315$. Let $2 \neq p \in \pi(G)$, by $(*)$, $p \mid (1+m_p)$ and $(p-1) \mid m_p$, which implies that $p \in \{3, 5, 7, 19, 2689, 5041\}$.

If $19 \in \pi(G)$, then $m_{19} = 3780$. On the other hand, by $(*)$ we conclude that if $38 \in \pi_e(G)$, then $m_{38} \in \{5760, 3780, 5040\}$ and $38 \mid (1 + m_2 + m_{19} + m_{38})$, a contradiction. Therefore $38 \notin \pi_e(G)$. Thus the group P_{19} acts fixed point freely on the set of elements of order 2, and $|P_{19}| \mid m_2$, which is a contradiction. Hence $19 \notin \pi(G)$. Similar to the above discussion 2689, $5041 \notin \pi(G)$, and so $\pi(G) \subseteq \{2, 3, 5, 7\}$.

If 3, 5 and $7 \in \pi(G)$, then $m_3 = 1232$, $m_5 = 1344$ and $m_7 = 5760$, by $(*)$. Also we can see easily that G does not contain any elements of order 35 , 512, 81, 125, 343 and 768.

If 15, 25, $49 \in \pi_e(G)$, then $m_{15} = 2688$, $m_{25} = 3780$ and $m_{49} = 1344$.

If $2^a \times 3^b \in \pi_e(G)$, then $1 \leq a \leq 6$ and $1 \leq b \leq 4$.

If $2^c \times 5^d \in \pi_e(G)$, then $1 \le c \le 6$ and $1 \le d \le 2$.

If $2^e \times 7^f \in \pi_e(G)$, then $1 \le e \le 7$ and $1 \le f \le 2$. If $3^k \times 5^h \in \pi_e(G)$, then $1 \le k \le 3$ and $1 \le h \le 2$.

If $3^l \times 7^m \in \pi_e(G)$, then $1 \le l \le 3$ and $1 \le m \le 2$.

We show that $\pi(G)$ could not be the sets $\{2\}$, $\{2, 3\}$ and $\{2, 3, 5\}$ and $\{2, 3, 7\}$, and $\pi(G)$ must be equal to $\{2, 3, 5, 7\}$.

Case a. If $\pi(G) = \{2\}$, then $\pi_e(G) \subseteq \{1, 2, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8\}$ and $|G| = 2^m = 20160 + 1232k_1 + 1344k_2 + 2688k_3 + 3780k_4 + 5040k_5 + 5760k_6$, where m, k_1, k_2, k_3, k_4, k_5 and k_6 are non-negative integers and $0 \le k_1 + k_2 + k_3 + k_4 + k_5 + k_6 \le 1$. It easy to check that this equation has no solution.

Case b. We claim that $3 \in \pi(G)$. Suppose contrary, i.e, $3 \notin \pi(G)$. If 5, $7 \notin \pi(G)$, then by Case a, we get a contradiction. Hence 5 or $7 \in \pi(G)$.

Let $5 \in \pi(G)$. Since $125 \notin \pi_e(G)$, $\exp(P_5) = 5$ or 25.

If $\exp(P_5) = 5$, then $|P_5| \mid (1 + m_5) = 1345$. Hence $|P_5| = 5$, and so $n_5 = 1344/4 \mid |G|$. Because $3 \notin \pi(G)$, we get a contradiction.

If $\exp(P_5) = 25$, then $|P_5| \mid 125$. Suppose that $|P_5| = 25$, then $n_5 = 3780/20 \mid |G|$, a contradiction. If $|P_5| = 125$, then by Lemma 2.2, $25 \mid m_{25}$, a contradiction. Thus $5 \notin \pi(G)$.

Let $7 \in \pi(G)$. Since $7^3 \notin \pi_e(G)$, $\exp(P_7) = 7$ or 49.

If $\exp(P_7) = 7$, then $|P_7| = 7$ and $n_7 = m_7/6 = 5760/6 \mid |G|$, a contradiction.

If $\exp(P_7) = 49$, then $|P_7| = 49$. Hence $n_7 = m_{49}/42 = 32$ and by Sylow's theorem, we get a contradiction.

Case c. Let $\pi(G) = \{2, 3\}$. Since $3^4 \notin \pi_e(G)$, $\exp(P_3) = 3$, 3^2 or 3^3. If $\exp(P_3) = 3$, then $|P_3| \mid (1 + m_3) = 1233$. Hence $|P_3| \mid 9$. Thus $|P_3| = 3$ or 9. First suppose that $|P_3| = 3$. Then $n_3 = m_3/2 = 1232/2 \mid |G|$, a contradiction. Suppose that $|P_3| = 9$. Thus $|G| = 2^m \times 9 = 20160 + 1232k_1 + 1344k_2 + 2688k_3 + 3780k_4 + 5040k_5 + 5760k_6$, where m, k_1, k_2, k_3, k_4, k_5 and k_6 are non-negative integers and $0 \le k_1 + k_2 + k_3 + k_4 + k_5 + k_6 \le 9$. It is clear that $|G| \le 72000$. Since $20160 \le |G| \le 72000$, $m = 12$. Therefore $16704 = 1232k_1 + 1344k_2 + 2688k_3 + 3780k_4 + 5040k_5 + 5760k_6$. By using an easy computer calculation, we can see that this equation has no solution.

Let $\exp(P_3) = 9$. Since $m_9 \in \{3780, 5760\}$ and $|P_3| \mid (1 + m_3 + m_9)$, $|P_3| = 9$. Hence $n_3 = m_9/6 \mid |G|$, which is a contradiction.

Let $\exp(P_3) = 27$. Since $m_{27} \in \{3780, 5760\}$ and $|P_3| \mid (1+m_3+m_9+m_{27})$, $|P_3| \mid 81$. If $|P_3| = 81$, then by Lemma 2.2, $27 \mid m_{27}$, which is a contradiction. If $|P_3| = 27$, then $n_3 = m_{27}/18 \mid |G|$, a contradiction.

Case d. Let $\pi(G) = \{2, 3, 5\}$. Since $5^3 \notin \pi_e(G)$, $\exp(P_5) = 5$ or 25. If $\exp(P_5) = 5$, then $|P_5| \mid (1 + m_5) = 1345$. Hence $|P_5| = 5$, and $n_5 = 1344/4 \mid |G|$, a contradiction. If $\exp(P_5) = 25$, then $|P_5| \mid (1 + m_5 + m_{25})$. Thus $|P_5| \mid 125$ and $|P_5| = 25$ or 125. If $|P_5| = 25$, then $n_5 = 3780/20 \mid |G|$, a contradiction. If $|P_5| = 125$, then $25 \mid m_{25}$, a contradiction.

Case e. Let $\pi(G) = \{2, 3, 7\}$. Since $7^3 \notin \pi_e(G)$, $\exp(P_7) = 7$ or 49. If $\exp(P_7) = 7$, then $|P_7| \mid (1 + m_7)$. Thus $|P_7| = 7$ and $n_7 = 960$. Since $n_7 \mid |G|$ and $5 \notin \pi(G)$, we get a contradiction.

If $\exp(P_7) = 49$, then $|P_7| \mid (1 + m_7 + m_{49})$. Thus $|P_7| = 49$ and $n_7 = 32$. By Sylow's theorem, we get a contradiction.

Case f. Let $\pi(G) = \{2, 3, 5, 7\}$. Since $35 \notin \pi_e(G)$, the group P_7 acts fixed point freely on the set of elements of order 5, and so $|P_7| \mid m_5 = 1344$, which implies that $|P_7| = 7$. Similarly, we can conclude that $|P_5| = 5$. We have $m_{21} = \phi(21) \cdot n_7 \cdot k$, where k is the number of cyclic subgroups of order 3 in $C_G(P_7)$. Since $n_7 = m_7/\phi(7) = 960$, $2 \times 5760 \mid m_{21}$, a contradiction. Hence $21 \notin \pi_e(G)$. Similarly, $10 \notin \pi_e(G)$.

Since $21 \notin \pi_e(G)$, the group P_3 acts fixed point freely on the set of elements of order 7. Then $|P_3| \mid m_7 = 5760$. Thus $|P_3| \mid 9$. Also since $10 \notin \pi_e(G)$, $|P_2| \mid m_5 = 1344$, and so $|P_2| \mid 2^6$. If $|P_3| = 3$, then $|G| = 2^m \times 105$. On the other hand, since $|P_2| \mid 2^6$, $m \leq 6$. Since $|G| \leq 20160$, $2^m \times 105 \leq 20160$, a contradiction. Therefore $|P_3| = 9$ and then $|G| = |A_8|$. By [4], since A_8 is a simple K_4-group, $G \cong A_8$, and the proof is complete.

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Alireza Khalili Asboei,
Department of Mathematics,
Science and Research Branch,
Islamic Azad University, Tehran, Iran.
Email: khaliliasbo@yahoo.com

Syyed Sadegh Salehi,
Department of Mathematics,
Science and Research Branch,
Islamic Azad University, Tehran, Iran.
Email: salehissss@yahoo.com

Ali Iranmanesh,
Department of Mathematics,
Tarbiat Modares University
P. O. Box: 14115-137, Tehran, Iran.
Email: iranmanesh@modares.ac.ir

Abolfazl Tehranian,
Department of Mathematics,
Science and Research Branch,
Islamic Azad University, Tehran, Iran.
Email: tehranian1340@yahoo.com