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Characterization of Speherical Helices in Euclidean 3-Space

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Abstract

In this paper, we calculate Frenet frames of the *tangent indicatrix* (t), *principal normal indicatrix* (n) and *binormal indicatrix* (b) of the curve α in $\mathbb{R}^3$ which are spherical curves. Also, we give some differential equations which are characterizations for (t), (n) and (b) to be general helix. Morever we give a characterization for *tangent indicatrix* (t) to be a circle.

1 Introduction

In differential geometry, we think of curves as a geometric set of points of locus. Curves theory is a important workframe in the diferential geometry studies and we have a lot of spacial curve. Helix is one of these spacial curves. We can see helical structures in nature and mechanic tools [2, 8]. In the field of computer aided design and computer graphics, helices can be used for the tool path description, the simulation of kinematic motion or design of highways, etc. [10]. Also we can see the helix curve or helical structure in fractal geometry, for instance hyperhelices [3]. In differential geometry; a curve of constant slope or general helix in Euclidean 3-space $\mathbb{R}^3$, is defined by the property that the tangent makes a constant angle with a fixed straight line (the axis of the general helix). A classical result stated by *M. A. Lancret* in 1802 and first proved by *B. de Saint Venant* in 1845 (see [5, 9] for details) is: A necessary

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and suficient condition that a curve be a general helix is that the ratio of curvature to torsion be constant. If both of $\varkappa$ and τ are non-zero constant it is, of course, a general helix. We call it a circular helix. Its known that straight line $(\kappa(s) = 0)$ and circle $(\tau(s) = 0)$ are degenerate-helix examples.

In [6], Kula and Yayli studied on slant helix and its spherical indicatrix in $\mathbb{R}^3$.

In [7], Kula, Ekmekci, Yaylı and İlarslan studied characterizations of slant helices in Euclidean 3-space.

In this paper, we consider to be general helix of the tangent indicatrix (t), principal normal indicatrix (n) and binormal indicatrix (b) of the curve α in $\mathbb{R}^3$. We obtain some differantial equations which are characterizations of a general helix using the Frenet frames of the tangent indicatrix (t), principal normal indicatrix (n) and binormal indicatrix (b) of re-parameterize curve α.

2 Preliminaries

In this section, we give some basic consepts on classical differantial geometry of space curve and the definations of general helix and the Frenet frame of the $(t),(n)$ and (b) in $\mathbb{R}^3$. Let $\alpha : I \subset \mathbb{R} \longrightarrow \mathbb{R}^3$ be a unit speed curve *with arc lenght parameter* s. α is a general helix if there is some constant vector u, so that $t.u = \cos\theta$ is constant along the curve, where $t(s) = \alpha'(s)$ is a unit tangent vector of α at s. We define the curvature of α by $\kappa(s) = \|\alpha''(s)\|$. If $\kappa(s) \neq 0$, then the unit principal normal vector $n(s)$ of the curve α at s is given by $\alpha''(s) = \kappa(s)n(s)$. The unit vector $b(s) = t(s) \times n(s)$ called the unit binormal vector of α at s. Then Frenet equations for α are given by

$$\begin{bmatrix} t'(s) \\ n'(s) \\ b'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{bmatrix} \begin{bmatrix} t(s) \\ n(s) \\ b(s) \end{bmatrix}$$

where $\tau(s)$ is the torsion of the curve α at s. It is known that curve α is a general helix if and only if $\left(\frac{\tau}{\kappa}\right)(s)$ =constant.

Definition 2.1. Let α be a unit speed regular curve in Euclidean 3-space with Frenet vectors t , n and b. The unit tangent vectors along the curve α generate a curve (t) on the sphere of radius 1 about the origin. The curve (t) is called the spherical indicatrix of t or more commonly, (t) is called tangent indicatrix of the curve α. If $\alpha = \alpha(s)$ is a natural representation of α, then $(t) = t(s)$ will be a representation of (t). Similarly one considers the principal normal indicatrix $(n) = n(s)$ and binormal indicatrix $(b) = b(s)$.

In this paper, by D we denote the covariant differentiation of $\mathbb{R}^3$. The Serret-Frenet formulae for $(t),(n)$ and (b) in $\mathbb{R}^3$ are follows:

Remark 2.2. If the Frenet frame of the tanget indicatrix (t) of a space curve α is $\{\mathsf{T},\mathsf{N},\mathsf{B}\}$ and parameterzied by the arclenght $s_t=\int \kappa(s)ds$, then we have Serret-Frenet equations:

$$\begin{bmatrix} D_{\mathsf{T}}\mathsf{T} \\ D_{\mathsf{T}}\mathsf{N} \\ D_{\mathsf{T}}\mathsf{B} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & f_t \\ 0 & -f_t & 0 \end{bmatrix} \begin{bmatrix} \mathsf{T} \\ \mathsf{N} \\ \mathsf{B} \end{bmatrix} \tag{2.1}$$

where $f_t=\frac{\tau_t}{\kappa_t}$is curvature of the *tangent indicatrix* (t) [1].

Remark 2.3. If the Frenet frame of the *principal normal indicatrix* (n) of a space curve α is $\{\mathcal{T},\mathcal{N},\mathcal{B}\}$ and parameterzied by the arclenght $s_n=\int\sqrt{\kappa^2(s)+\tau^2(s)}ds$, then we have Serret-Frenet equations:

$$\begin{bmatrix} D_{\mathcal{T}}\mathcal{T} \\ \\ D_{\mathcal{T}}\mathcal{N} \\ \\ D_{\mathcal{T}}\mathcal{B} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{1+f_n^2}} & 0 \\ -\frac{1}{\sqrt{1+f_n^2}} & 0 & \frac{f_n}{\sqrt{1+f_n^2}} \\ 0 & -\frac{f_n}{\sqrt{1+f_n^2}} & 0 \end{bmatrix} \begin{bmatrix} \mathcal{T} \\ \\ \mathcal{N} \\ \\ \mathcal{B} \end{bmatrix} \tag{2.2}$$

or $\left(F=\frac{1}{\sqrt{1+f_n^2}}, G=\frac{f_n}{\sqrt{1+f_n^2}}\right)$

$$\begin{bmatrix} D_{\mathcal{T}}\mathcal{T} \\ D_{\mathcal{T}}\mathcal{N} \\ D_{\mathcal{T}}\mathcal{B} \end{bmatrix} = \begin{bmatrix} 0 & F & 0 \\ -F & 0 & G \\ 0 & -G & 0 \end{bmatrix} \begin{bmatrix} \mathcal{T} \\ \mathcal{N} \\ \mathcal{B} \end{bmatrix} \tag{2.3}$$

where $f_n=\frac{\tau_n}{\kappa_n}$is curvature of the *principal normal indicatrix* (n).

Remark 2.4. If the Frenet frame of the *binormal indicatrix* (b) of a space curve α is $\{\mathbb{T},\mathbb{N},\mathbb{B}\}$ and parameterzied by the arclenght $s_b=\int\tau(s)ds$, then we have Serret-Frenet equations:

$$\begin{bmatrix} D_{\mathbb{T}}\mathbb{T} \\ D_{\mathbb{T}}\mathbb{N} \\ D_{\mathbb{T}}\mathbb{B} \end{bmatrix} = \begin{bmatrix} 0 & f_b & 0 \\ -f_b & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{T} \\ \mathbb{N} \\ \mathbb{B} \end{bmatrix} \tag{2.4}$$

where $f_b=\frac{\kappa_b}{\tau_b}$is curvature of the *binormal indicatrix* (b).

3 Characterizations for Tangent, Normal and Binormal Indicatrix

In this section, we give some characterizations to be a general helix for *tangent indicatrix* (t), *principal normal indicatrix* (n) and *binormal indicatrix* (b) and to be a circle *tangent indicatrix* (t) of a unit speed curve α in $\mathbb{R}^3$, respectively.

Theorem 3.1. *Let α be a unit speed smooth curve with arc lenght parameter s. Its tangent indicatrix (t) is a general helix if and only if tangent vector field T of the tangent indicatrix of the curve α is providing the following equation:*

$$D_{\mathsf{T}}^3\mathsf{T}+(1+f_t^2)D_{\mathsf{T}}\mathsf{T}=0, \tag{3.1}$$

where f_t is curvature of the tangent indicatrix (t) of the curve α.

Proof. Suppose that *tangent indicatrix* (t) of the curve α be a general helix. Thus, its curvature $f_t = \frac{\tau_t}{\kappa_t}$ must be non-zero constant. And so $f_t^{\prime} = 0$.

From (2.1), we have $D_{\mathsf{T}}\mathsf{T} = \mathsf{N}$. By differentiating $D_{\mathsf{T}}\mathsf{T} = \mathsf{N}$, we get

$$D_{\mathsf{T}}^3\mathsf{T} + (1+f_t^2)\mathsf{N} = 0. \tag{3.2}$$

We have from Frenet frame of *tangent indicatrix* (t) of the curve α

$$D_{\mathsf{T}}\mathsf{T} = \mathsf{N}. \tag{3.3}$$

Using equations (3.2) and (3.3), we get

$$D_{\mathsf{T}}^3\mathsf{T} + (1+f_t^2)D_{\mathsf{T}}\mathsf{T} = 0.$$

Conversely let us assume that (3.1) holds. From (2.1), we have $D_{\mathsf{T}}\mathsf{T} = \mathsf{N}$. By differentiating $D_{\mathsf{T}}\mathsf{T} = \mathsf{N}$, we get

$$D_{\mathsf{T}}^3\mathsf{T} + (1+f_t^2)\mathsf{N} - f_t^{\prime}\mathsf{B} = 0. \tag{3.4}$$

From (2.1), we get

$$\mathsf{B} = \frac{1}{f_t}\left\{D_{\mathsf{T}}^2\mathsf{T} + \mathsf{T}\right\}. \tag{3.5}$$

Using equations (3.3), (3.4) and (3.5)

$$D_{\mathsf{T}}^3\mathsf{T} - \frac{f_t^{\prime}}{f_t}D_{\mathsf{T}}^2\mathsf{T} + (1+f_t^2)D_{\mathsf{T}}\mathsf{T} - \frac{f_t^{\prime}}{f_t}\mathsf{T} = 0, \tag{3.6}$$

and using equations (3.1) and (3.6)

$$f_t^{\prime} = 0 \text{ and } \frac{\tau_t}{\kappa_t} = c \text{ (non-zero constant)}$$

Thus, *tangent indicatrix* (t) of the curve α must be a general helix. $\square$

Theorem 3.2. *Let α be a unit speed smooth curve with arc lenght parameter s. Its tangent indicatrix (t) is a circle if and only if tangent vector field T of the tangent indicatrix of the curve α is providing the following equation:*

$$D_{\mathsf{T}}^2\mathsf{T} + \mathsf{T} = 0. \tag{3.7}$$

Proof. If *tangent indicatrix* (t) of the curve α is a circle, its $\tau_t = 0$. And so $f_t = 0$.

From (2.1), we have $D_{\mathsf{T}}\mathsf{T} = \mathsf{N}$. By differentiating $D_{\mathsf{T}}\mathsf{T} = \mathsf{N}$, we get

$$D_{\mathsf{T}}^2\mathsf{T} + \mathsf{T} = 0.$$

Conversely let us assume that (3.7) holds. From (2.1), we have $D_{\mathsf{T}}\mathsf{T} = \mathsf{N}$. By differentiating $D_{\mathsf{T}}\mathsf{T} = \mathsf{N}$, we get

$$D_{\mathsf{T}}^2\mathsf{T} + \mathsf{T} - f\mathsf{B} = 0 \tag{3.8}$$

Using equations (3.7) and (3.8), we get

$$f = 0 \text{ and } \frac{\tau_t}{\kappa_t} = 0 \tag{3.9}$$

From equation (3.9), we can say $\tau_t = 0$. Thus *tangent indicatrix* (t) is a circle. □

Theorem 3.3. *Let α be a unit speed smooth curve with arc lenght parameter s. Its tangent indicatrix (t) is a general helix if and only if binormal vector field B of the tangent indicatrix of the curve α is providing the following equation:*

$$D_{\mathsf{T}}^3\mathsf{B} + (1 + f_t^2)D_{\mathsf{T}}\mathsf{B} = 0, \tag{3.10}$$

where f_t is curvature of the tangent indicatrix (t) of the curve α.

Proof. Suppose that *tangent indicatrix* (t) of the curve α be a general helix. Thus, its curvature $f_t = \frac{\tau_t}{\kappa_t}$ must be non-zero constant. And so $f_t^{\prime} = 0$.

From (2.1), we have $D_{\mathsf{T}}\mathsf{B} = -f_t\mathsf{N}$. By differentiating $D_{\mathsf{T}}\mathsf{B} = -f_t\mathsf{N}$, we get

$$D_{\mathsf{T}}^3\mathsf{B} - f_t\mathsf{N} + f_t^2 D_{\mathsf{T}}\mathsf{B} = 0. \tag{3.11}$$

From (2.1), we get

$$\mathsf{N} = -\frac{1}{f_t}D_{\mathsf{T}}\mathsf{B} \tag{3.12}$$

Using equations (3.11) and (3.12), we get

$$D_{\mathsf{T}}^3\mathsf{B} + (1 + f_t^2)D_{\mathsf{T}}\mathsf{B} = 0.$$

Conversely let us assume that (3.10) holds. From (2.1), we have $D_{\mathsf{T}}\mathsf{B} = -f_t\mathsf{N}$. By differentiating $D_{\mathsf{T}}\mathsf{B} = -f_t\mathsf{N}$, we get

$$D_{\mathsf{T}}^3\mathsf{B} + \left(f_t^{\prime\prime} - f_t\right)\mathsf{N} - 2f_t^{\prime}\mathsf{T} + 3f_t^{\prime}f_t\mathsf{B} + f_t^2 D_{\mathsf{T}}\mathsf{B} = 0. \tag{3.13}$$

From (2.1), we get

$$\mathsf{T} = \frac{1}{f_t} D^2_{\mathsf{T}}\mathsf{B} + \frac{f_t^{\prime}}{f_t^2} D_{\mathsf{T}}\mathsf{B} + f_t \mathsf{B} \tag{3.14}$$

Using equations (3.12), (3.13) and (3.14), we get

$$D^3_{\mathsf{T}}\mathsf{B} - 2\frac{f_t^{\prime}}{f_t} D^2_{\mathsf{T}}\mathsf{B} + \left(1 + f_t^2 - \frac{f_t^{\prime\prime}}{f_t} - 2\left(\frac{f_t^{\prime}}{f_t}\right)^2\right) D_{\mathsf{T}}\mathsf{B} + f_t^{\prime} f_t \mathsf{B} = 0, \tag{3.15}$$

and using equations (3.10) and (3.15), we get

$$f_t^{\prime} = 0 \text{ and } \frac{\tau_t}{\kappa_t} = c_1 \text{ (non-zero constant).}$$

Thus, *tangent indicatrix* (t) of the curve α must be a general helix. □

Theorem 3.4. *Let α be a unit speed smooth curve with arc lenght parameter s. Its principal normal indicatrix (n) is a general helix if and only if normal vector field $\mathcal{N}$ of the principal normal indicatrix of the curve α is providing the following equation:*

$$D^2_{\mathcal{T}}\mathcal{N} + \mathcal{N} = 0. \tag{3.16}$$

Proof. Suppose that *principal normal indicatrix* (n) of the curve α be a general helix. Thus, it is curvature $f_n = \frac{\tau_n}{\kappa_n}$ must be non-zero constant. And so $f_n^{\prime} = 0$. From (2.3), we have $D_{\mathcal{T}}\mathcal{N} = -F\mathcal{T} + G\ \mathcal{B}$. By differentiating $D_{\mathcal{T}}\mathcal{N} = -F\mathcal{T} + G$ $\mathcal{B}$, we get

$$D^2_{\mathcal{T}}\mathcal{N} + \left(F^2 + G^2\right)\mathcal{N} + F^{\prime}\mathcal{T} - G^{\prime}\mathcal{B} = 0 \tag{3.17}$$

If we calculate $F^{\prime}$, $G^{\prime}$ and $F^2 + G^2$, we find

$$F^{\prime} = \frac{f_n f_n^{\prime}}{(1 + f_n^2)^{3/2}},\ G^{\prime} = \frac{f_n^{\prime}}{(1 + f_n^2)^{3/2}} \tag{3.18}$$

and

$$F^2 + G^2 = 1. \tag{3.19}$$

Using equations (3.17), (3.18), (3.19) and $f_n^{\prime} = 0$, we get

$$D^2_{\mathcal{T}}\mathcal{N} + \mathcal{N} = 0.$$

Conversely let us assume that (3.16) holds. From (2.3), we have $D_{\mathcal{T}}\mathcal{N} = -F\mathcal{T} + G\ \mathcal{B}$. By differentiating $D_{\mathcal{T}}\mathcal{N} = -F\mathcal{T} + G\ \mathcal{B}$, we get

$$D^2_{\mathcal{T}}\mathcal{N} + \left(F^2 + G^2\right)\mathcal{N} + F^{\prime}\mathcal{T} - G^{\prime}\mathcal{B} = 0.$$

We know $F^2+G^2=1$ so we can write from last equation

$$D_{\mathcal{T}}^2\mathcal{N}+\mathcal{N}+F^{\shortmid}\mathcal{T}-G^{\shortmid}\mathcal{B}=0. \tag{3.20}$$

From Serret-Frenet equations of *principal normal indicatrix* (n), we have

$$F^{\shortmid}\mathcal{T}-G^{\shortmid}\mathcal{B}=-\frac{f_n^{\shortmid}}{f_n\left(1+f_n^2\right)^{3/2}}D_{\mathcal{T}}\mathcal{N}. \tag{3.21}$$

Using aquations (3.20) and (3.21), we get

$$D_{\mathcal{T}}^2\mathcal{N}-\frac{f_n^{\shortmid}}{f_n\left(1+f_n^2\right)^{3/2}}D_{\mathcal{T}}\mathcal{N}+\mathcal{N}=0, \tag{3.22}$$

and using equations (3.16) and (3.22),

$$f_n^{\shortmid}=0 \text{ and } \frac{\tau_n}{\kappa_n}=c_2 \text{ (non-zero constant).}$$

Thus, *principal normal indicatrix* (n) of the curve α must be a general helix. □

Theorem 3.5. *Let α be a unit speed smooth curve with arc lenght parameter s. Its binormal indicatrix (b) is a general helix if and only if binormal vector field $\mathbb{B}$ of the binormal indicatrix of the curve α is providing the following equation:*

$$D_{\mathbb{T}}^3\mathbb{B}+(1+f_b^2)D_{\mathbb{T}}\mathbb{B}=0. \tag{3.23}$$

where f_b is curvature of the binormal indicatrix (b) of the curve α.

Proof. Suppose that *binormal indicatrix* (b) of the curve α be a general helix. Thus, its curvature $f_b=\frac{\kappa_b}{\tau_b}$ must be non-zero constant. And so $f_b^{\shortmid}=0$.

From (2.4), we have $D_{\mathbb{T}}\mathbb{B}=-\mathbb{N}$. By differentiating $D_{\mathbb{T}}\mathbb{B}=-\mathbb{N}$, we get

$$D_{\mathbb{T}}^3\mathbb{B}-f_b^2\mathbb{N}+D_{\mathbb{T}}\mathbb{B}=0. \tag{3.24}$$

From (2.4), we get

$$\mathbb{N}=-D_{\mathbb{T}}\mathbb{B} \tag{3.25}$$

Using equations (3.24) and (3.25), we get

$$D_{\mathbb{T}}^3\mathbb{B}+(1+f_b^2)D_{\mathbb{T}}\mathbb{B}=0.$$

Conversely let us assume that (3.23) holds. From (2.4), we have $D_{\mathbb{T}}\mathbb{B}=-\mathbb{N}$. By differentiating $D_{\mathbb{T}}\mathbb{B}=-\mathbb{N}$, we get

$$D_{\mathbb{T}}^3\mathbb{B}-f_b^{\shortmid}\mathbb{T}-f_b^2\mathbb{N}+D_{\mathbb{T}}\mathbb{B}=0. \tag{3.26}$$

From (2.4), we have

$$\mathbb{T} = \frac{1}{f_b}\left(D_{\mathbb{T}}^2\mathbb{B} + \mathbb{B}\right). \tag{3.27}$$

Using equations (3.25), (3.26) and (3.27), we get

$$D_{\mathbb{T}}^3\mathbb{B} - \frac{f_b^{'}}{f_b}D_{\mathbb{T}}^2\mathbb{B} + (1+f_b^2)D_{\mathbb{T}}\mathbb{B} - \frac{f_b^{'}}{f_b}\mathbb{B} = 0 \tag{3.28}$$

and using equations (3.23) and (3.28),

$$f_b^{'} = 0 \text{ and } \frac{\kappa_b}{\tau_b} = c_3 \text{ (non-zero constant).}$$

Thus, *binormal indicatrix* (b) of the curve α must be a general helix. □

Theorem 3.6. *Let α be a unit speed smooth curve with arc lenght parameter s. Its binormal indicatrix (b) is a general helix if and only if tangent vector field $\mathbb{T}$ of the binormal indicatrix of the curve α is providing the following equations:*

$$D_{\mathbb{T}}^3\mathbb{T} + (1+f_b^2)D_{\mathbb{T}}\mathbb{T} = 0. \tag{3.29}$$

where f_b is curvature of the binormal indicatrix (b) of the curve α.

Proof. Suppose that *binormal indicatrix* (b) of the curve α be a general helix. Thus, its curvature $f_b = \frac{\kappa_b}{\tau_b}$ must be non-zero constant. And so $f_b^{'} = 0$.

From (2.4), we have $D_{\mathbb{T}}\mathbb{T}=f_b\mathbb{N}$. By differentiating $D_{\mathbb{T}}\mathbb{T}=f_b\mathbb{N}$, we get

$$D_{\mathbb{T}}^3\mathbb{T} + f_b\mathbb{N} + f_b^2D_{\mathbb{T}}\mathbb{T} = 0. \tag{3.30}$$

From (2.4), we get

$$\mathbb{N} = \frac{1}{f_b}D_{\mathbb{T}}\mathbb{T} \tag{3.31}$$

Using equations (3.30) and (3.31), we get

$$D_{\mathbb{T}}^3\mathbb{T} + (1+f_b^2)D_{\mathbb{T}}\mathbb{T} = 0.$$

Conversely let us assume that (3.29) holds. From (2.4), we have $D_{\mathbb{T}}\mathbb{T}=f_b\mathbb{N}$. By differentiating $D_{\mathbb{T}}\mathbb{T}=f_b\mathbb{N}$, we get

$$D_{\mathbb{T}}^3\mathbb{T} + \left(f_b - f_b^{''}\right)\mathbb{N} + 3f_b^{'}f_b\mathbb{T} - 2f_b^{'}\mathbb{B} + f_b^2D_{\mathbb{T}}\mathbb{T} = 0. \tag{3.32}$$

From (2.4), we get

$$\mathbb{B} = \frac{1}{f_b}D_{\mathbb{T}}^2\mathbb{T} - \frac{f_b^{'}}{f_b^2}D_{\mathbb{T}}\mathbb{T} \tag{3.33}$$

Using equations (3.31), (3.32) and (3.33), we get

$$D_{\mathbb{T}}^{3}\mathbb{T}-2\frac{f_b^{'}}{f_b}D_{\mathbb{T}}^{2}\mathbb{T}+\left(1+f_b^2+2\left(\frac{f_b^{'}}{f_b}\right)^2-\frac{f_b^{''}}{f_b}\right)D_{\mathbb{T}}\mathbb{T}+3f_b^{'}f_b\mathbb{T}=0, \qquad (3.34)$$

and using equations (3.29) and (3.34)

$$f_b^{'}=0 \text{ and } \frac{\kappa_b}{\tau_b}=c_4 \text{ (non-zero constant).}$$

Thus, *binormal indicatrix* (b) of the curve α must be a general helix. □

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